Name: Solutions **Section**: ____________

Clear your desk of everything excepts pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

1. (2 points) Suppose $\int_{-1}^{2} f(x)dx = 5$ and $\int_{-1}^{3} f(x)dx = 7$. Find $\int_{2}^{3} 5\ f(x)dx$.

   By integral properties we know that:

$$\int_{-1}^{3} f(x)dx = \int_{-1}^{2} f(x)dx + \int_{2}^{3} f(x)dx.$$

   Thus, $\int_2^3 f(x)dx = 7 - 5 = 2$ and

$$\int_{2}^{3} 5f(x)dx = 5\int_{2}^{3} f(x)dx = \underline{10}.$$

2. (2 points) Let $F(x) = \int_{\cos x}^{1} \sqrt{1-t^2}\ dt$. Find $F'(x)$.

$$\begin{aligned}
F'(x) &= \frac{d}{dx}\int_{\cos x}^{1} \sqrt{1-t^2}\ dt \\
&= -\frac{d}{dx}\int_{1}^{\cos x} \sqrt{1-t^2}\ dt && \text{(Flip integration bounds)} \\
&= \underline{-\sqrt{1-(\cos x)^2}\ (-\sin x)} && \text{(Use FTC I and chain rule)} \\
&= \underline{\sin^2 x} && \text{(Simplify/not a neccesary step)}
\end{aligned}$$

3. (2 points) Evaluate: $\int_{1}^{2} \frac{t^3+\sqrt{t}}{t}\ dt$.

$$\begin{aligned}
\int_{1}^{2} \frac{t^3+\sqrt{t}}{t}\ dt &= \int_{1}^{2} t^2 + t^{-1/2}dt && \text{(simplify)} \\
&= \int_{1}^{2} t^2 dt + \int_{1}^{2} t^{-1/2}dt && \text{(integral property)} \\
&= \frac{1}{3}t^3 \Big|_1^2 + 2t^{1/2}\Big|_1^2 && \text{(FTC II)} \\
&= \underline{\frac{7}{3} + 2(\sqrt{2}-1)}.
\end{aligned}$$

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4. A particle is moving with an acceleration given by $a(t) = \sin t$ (with units m/s^2).

(a) (2 points) Given that the initial velocity is $v(0) = 1$, find $v(\pi)$ the velocity at time $t = \pi$.

We can rewrite this as an Initial Value Problem: Find $v(t)$ where

$$\frac{dv}{dt} = \sin t, \qquad v(0) = 1.$$

The most-general solution is: $v(t) = -\cos t + C$.
Solve for C: $v(0) = -\cos 0 + C = 1 \implies \quad C = 2$.

$$v(t) = -\cos t + 2; \qquad v(\pi) = -\cos \pi + 2 = \underline{3\text{ m/s}}.$$

(b) (2 points) Given that the initial position is $x(0) = 0$, find $x(\pi)$ the position at time $t = \pi$.

We can rewrite this as an Initial Value Problem: Find $x(t)$ where

$$\frac{dx}{dt} = -\cos t + 2; \qquad x(0) = 0.$$

The most-general solution is: $x(t) = -\sin t + 2t + C$.
Solve for C: $x(0) = -\sin 0 + 2(0) + C = 0 \implies C = 0$.

$$x(t) = -\sin t + 2t; \qquad x(\pi) = -\sin \pi + 2\pi = \underline{2\pi\text{ m}}.$$