MTH 132: Exam 2 Study Guide

Instructor: Matthew Cha

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Main topics: Only a supplement for your study and NOT meant to be a complete list of topics. Make sure you know the precise textbook definitions and formulas. Some examples are given; these are taken from the textbook, lectures, quizzes and old exams.

Curve Sketching: (Sections 3.1–5)

1. A *critical point* is a number c in the domain of $f(x)$ such that either $f'(c)$ D.N.E. or $f'(c) = 0$.
2. Extreme Value Theorem: $f(x)$ continuous on $[a, b]$ has absolute maximum and absolute minimum.
3. Closed Interval Strategy: $f(x)$ is continuous on $[a, b]$ (Know how to find the domain of functions).
   i.) Find the domain and critical points. Compute the value of $f(x)$ at critical points.
   ii.) Find $f(a)$ and $f(b)$ (check the endpoint values).
   iii.) Compare all values: Largest is the *absolute maximum* and smallest is the *absolute minimum*.
   e.g. – Find the absolute max and min of $f(x) = x^3 - 12x$ on $[-5, 2]$.
   – Find the domain and critical points of $f(x) = \dfrac{x^2 - 4}{x^2 - x - 2}$; $f'(x) = \dfrac{x - 2}{x + 1}$.
4. Mean Value Theorem: (on formula sheet) Let $f(x)$ be continuous on $[a, b]$ and differentiable on (a, b). Then, there is a number c in (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.
5. Increasing/Decreasing Test: If $f'(x) > 0$ then $f(x)$ increasing. If $f'(x) < 0$ then $f(x)$ decreasing.
6. 1st Derivative Test: Let c be a critical point.
   i.) If $f'(x)$ changes from $+$ to $-$, then $f(c)$ is a local maximum.
   ii.) If $f'(x)$ changes from $-$ to $+$, then $f(c)$ is a local minimum.
   iii.) If $f'(x)$ does not change sign, then $f(c)$ is neither a local max or min.
7. Inflection points and concavity test: If $f''(x) > 0$ then concave up. If $f''(x) < 0$ then concave down.
8. Asymptotes:
   i.) Horizontal: e.g. $\lim_{x\to\infty} \dfrac{x^2 - 1}{3x^2 + x + 1} = \dfrac{1}{3}$; $\lim_{x\to\infty} \sqrt{x^2 + 1} - x = 0$.
   ii.) Vertical: e.g. $f(x) = \dfrac{x^2 - 4}{x^2 - x - 2}$ has only one vertical asymptote at $x = 1$.
   iii.) Slant: e.g. $f(x) = \dfrac{x^3 - 1}{x^2 - 4}$ (power on top is one bigger than bottom). Long division to compute.
9. Symmetry: *even* $f(x) = f(-x)$; *odd* $f(x) = -f(-x)$; *periodic* $f(x) = f(x + p)$.
   e.g. – Show that $f(x) = x \cos x$ is odd.
   – Show that $f(x) = \frac{x^2}{x^4 - 2x^2}$ is even.
10. Identify the graphs of $f(x)$, $f'(x)$ and $f''(x)$. e.g. Webwork 3.3.14, 2.7.11, 2.7.12

Applications of derivatives (Sections 2.9, 3.7, 3.8, 3.9)

1. Linear approximation: Linearization of $f(x)$ at $x = a$ is $L(x) = f'(a)(x-a) + f(a)$.
   i.) Near $x = a$ we can use linear approx. $L(x) \approx f(x)$.
   e.g. Use a linear approximation to estimate $\sqrt{17}$. (Ans:$\sqrt{17} \approx 4 + \frac{1}{8}$.)
   ii.) Differential: $\Delta f \approx df = f'(x)dx$ approximates the change in f.
   e.g. Use differentials to estimate change in $y = -\frac{1}{x}$ as x moves from 2 to 2.5. (Ans: $dy = \frac{1}{8}$.)
2. Optimization: Method to solve word problems.
   i.) Draw diagram and label quantities/variables that are changing.
   ii.) Write all relations/given information about variables. Eliminate variables.
   iii.) Apply closed interval strategy or 1st derivative test to find optimal values.
3. Newton's method: Given $f(x) = 0$ and x_1, then $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$. e.g. Use NM to approximate $\sqrt[5]{33}$.
4. Anti-derivatives: $F(x)$ is an anti-derivative of $f(x)$ if $F'(x) = f(x)$.
   i.) *Most-general* anti-derivative on an interval: $F(x) + C$
   ii.) Power rule: $f(x) = x^n$ for $n \neq -1$ then $F(x) = \frac{1}{n+1}x^{n+1}$.
   iii.) Trig derivatives to remember:
   $\frac{d}{dx}\sin x = \cos x$; $\frac{d}{dx}\cos x = -\sin x$; $\frac{d}{dx}\tan x = \sec^2 x$; $\frac{d}{dx}\sec x = \sec x \tan x$.
   iv.) Relationship between position, velocity and acceleration: $a(t) = \frac{d}{dt}v$, $v(t) = \frac{d}{dt}s$.
   $v(t)$ is an anti-derivative of $a(t)$. $s(t)$ is an anti-derivative of $v(t)$.
5. Initial Value Problems: e.g. Solve $\dfrac{df}{dx} = x^2 - 1$; $f(3) = -1$. Answer. $f(x) = \frac{1}{3}x^3 - x - 7$.

Areas, Sums and Integral (Sections 4.1, 4.2, 4.3)

1. Sigma notation: $\displaystyle\sum_{i=1}^{n} a_i = a_1 + a_2 + \ \ldots \ + a_{n-1} + a_n$. Also, know properties of sigma.
2. Left/Right Riemann Sums for $f(x)$ on $[a,b]$: Let $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$ then
   $R_n = \displaystyle\sum_{i=1}^{n} f(x_i)\Delta x$; $L_n = \displaystyle\sum_{i=1}^{n} f(x_{i-1})\Delta x$ (n = Number of rectangles used.)
   e.g. Estimate the net area of $f(x) = -x^2 + 8x + 9$ on $[0,8]$ using $n = 4$ rectangles.
3. Upper/Lower sums: For the height use the max/min in each interval
4. Summation formulas (on formula sheet): $\sum_{i=1}^{n} i$, $\sum_{i=1}^{n} i^2$.
5. Definite integral (net area): $f(x)$ continuous on $[a,b]$ then $\displaystyle\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$.
6. Use geometry to compute integrals: e.g $\displaystyle\int_0^a x\ dx = \frac{1}{2}a^2$ (triangle), $\int_{-1}^{1}\sqrt{1-x^2}\ dx = \pi/2$ (circle).
7. Know the integral properties: e.g $\displaystyle\int_a^b f(x)dx = -\int_b^a f(x)dx$; $\displaystyle\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx$.
8. Comparison properties: If $m \leq f(x) \leq M$ then $m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$.
9. Fundamental Theorem of Calculus:
   Part I: $\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$ e.g. Let $F(x) = \int_x^{x^4} t\sec t dt$ then $F'(x) = x^4\sec(x^4)(4x^3) - x\sec x$.
   Part II: If $f(x) = F'(x)$ then $\int_a^b f(x)dx = F(b) - F(a)$. e.g. $\int_0^1 \frac{t^3 - \sqrt{t}}{t}dt = \frac{1}{3} - 2$.