MTH 132: Exam 1 Study Guide

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1. Exam format:
6-8 Free response problems
6-8 Multiple choice problems
1 Challenge problem
Total points = 106 (up to 100 pts)

2. Test taking strategies:

- Read problem throughly, make sure to take note of what you are asked to find/compute.
- Show work! Be clear and logical in each of your steps. Communicate to the grader what you know and understand about the problem. This will ensure you receive maximal partial credit even if your make an error in calculation.
- Box/underline/circle your final answer. Include units if they are provided. In many cases, you do not need to simplify your final answer.

3. Main topics: Only a supplement for your study and NOT meant to be a complete list of topics. Make sure you know the precise textbook definitions and formulas. Some examples are given; these are taken from the textbook, lectures, quizzes and old exams.

Limits

1. Computing the limit of a function:
   i.) Direct substitution ii.) Limit laws iii.) Simplify, look for cancellation
   iv.) Identify limit by the graph the function
2. One-sided limits
   i.) Existence criterion: $\lim_{x\to a} f(x) = L$ if and only if $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = L$.
   ii.) Be comfortable with piecewise defined functions and absolute value functions,
   e.g. $\displaystyle\lim_{x\to 3^-} \frac{|x-3|}{x-3} = -1.$
3. Infinite limits and vertical asymptotes
   e.g. $\displaystyle\lim_{x\to 3^+} \frac{1}{x-3} = +\infty;$ $\displaystyle\lim_{x\to 3^-} \frac{1}{x-3} = -\infty;$
   $\displaystyle f(x) = \frac{x-3}{x^2-2x-3}$ has vertical asymptote only at $x = -1$.
4. Squeeze Theorem; know the statement and how to use it.
   e.g. Show that $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$
   Useful: $-1 \le \sin x \le 1;$ and $-1 \le \cos x \le 1.$
5. Trig limits (see formula sheet).
   e.g. Show that $\displaystyle\lim_{x\to 5} \frac{\sin(8x-40)}{x-5} = 8.$

Continuity

1. Definition: $f(x)$ is continuous at $x=a$ if $\lim_{x\to a} f(x) = f(a)$.
   - Use the definition to show that a function is continuous
   - Identify points of continuity by the graph of a function
   - Choose parameters so that a function is continuous

   e.g. Let $f(x) = \begin{cases} 3-x^2 & \text{if } x \le 0 \\ (x-1)^2 + a & \text{if } x > 0 \end{cases}$. Find a value for a so that f is continuous at $x=0$.

2. Functions that are continuous on their domain: polynomials, root, rational, trigonometric. Be able to find the domain and interval(s) of continuity.

   e.g. Let $f(x) = \dfrac{(x-3)\cos x}{x^2-2x-3}$. Find the interval of continuity for $f(x)$.

   Answer: $(-\infty,-1)\cup(-1,3)\cup(3,\infty)$.

3. Intermediate Value Theorem (given on formula sheet).

   e.g. Generic: Show that $f(x)=N$ has a solution:

   Steps: i.) Show that $f(x)$ is continuous on (a,b)

   ii.) Show that N is an intermediate value, e.g. $f(a)<N<f(b)$ or $f(b)<N<f(a)$.

   iii.) By the IVT there is a solution c such that $a<c<b$ and $f(c)=N$.

Derivatives

1. Sketch secant and tangent lines
2. Average rate of change: ARoC$(a \le x \le b) = \frac{f(b)-f(a)}{b-a}$. Relation to slope of the secant line.
3. Definition of derivate: $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$. Multiple meanings: instantaneous rate of change of $f(x)$ at $x=a$, slope of tangent line to $f(x)$ at point $x=a$

   Use the definition to compute the derivative of various functions e.g. x, $\frac{1}{x}$, $\sqrt{x}$.
4. Theorem: If $f'(a)$ exists then $f(x)$ is continuous at $x=a$.
5. Equation for the tangent line of $f(x)$ at the point (a,b): $y=f'(a)(x-a)+b$. (Also, $f(a)=b$.)
6. Derivative formulas: product rule, quotient rule, power rule, chain rule.
7. Derivative of trig functions:

   $\dfrac{d}{dx}\sin x = \cos x$; $\dfrac{d}{dx}\cos x = -\sin x$; $\dfrac{d}{dx}\tan x = \sec^2 x$; $\dfrac{d}{dx}\sec x = \sec x \tan x$.
8. Higher derivatives: e.g. If $f(x)=\sin(2x)$ find $f^{(20)}(x)$. Answer: $f^{(20)}(x) = 2^{20}\sin(2x)$.
9. Implicit differentiation: take the derivative on both sides of the equation, remember to use the chain rule. e.g. $\frac{d}{dx}y^2 = 2yy'$, $\frac{d}{dx}xy = y + xy'$.
10. Rates of change in sciences: e.g. position, velocity, acceleration examples. Include units!
11. Related rates: draw diagram, label and assign changing quantities by variables, find equations relating the variables and eliminate variables if possible, take implicit derivative, substitute known rate(s), quantities and solve for the unknown rate.

    e.g. Typical relations include: distance between points, Pythagorean theorem, similar triangles, area and volume formulas, etc.