1. For the following functions determine the amplitude, period, phase shift, domain, range. Then graph at least one complete cycle of the function, clearly label 5 consecutive “key points” and any vertical asymptotes. Be sure to label and scale the axes of your graphs.

- \( f(x) = 3 \cos(\pi x) - 7 \)
  - \( A = 3 \)
  - \( \text{per} = \frac{2\pi}{\pi} = 2 \)
  - \( \text{p.s.} = 0 \)
  - \( \text{domain} = (-\infty, \infty) \)
  - \( \text{range} = [-10, -4] \)

- \( g(x) = 5 \tan\left(2x + \frac{\pi}{4}\right) \)
  - \( A = 5 \)
  - \( \text{per} = \frac{\pi}{\frac{\pi}{2}} = \pi \)
  - \( \text{p.s.} = \left( \frac{1}{2} \cdot \frac{3\pi}{4} \right) = \frac{3\pi}{8} \)
  - \( \text{range} = (-\infty, \infty) \)

2. Name two consecutive vertical asymptotes for \( y = \cot\left(x - \frac{\pi}{6}\right) \)

3. Suppose a marble attached to the end of a spring is pulled down a distance of 8 inches from its resting position and then released. If the spring oscillates 3 times per second, find the model simple harmonic motion equation for the movement of the marble. What are the units of \( d \) and \( t \) in your equation?

- \( \text{freq} = 3 \) times per sec.
  - \( A = 8 \)
  - \( \text{per} = \frac{1}{\text{freq}} = \frac{1}{3} = \frac{2\pi}{w} \)
  - \( \frac{1}{2} \cdot w = 2\pi \Rightarrow w = 4\pi \)

- \( d = -8 \cos(6\pi t) \)
  - \( d = \text{inches} \) and \( t = \text{sec.} \)
4. Verify algebraically the following trig equations.

\[ \cot \left( \frac{\pi}{2} - \theta \right) \frac{\sec(-\theta) - (\sin^2 \theta + \cos^2 \theta)}{\cos \theta + \cot \theta} = \csc \theta + \cot \theta \]

\[
\begin{align*}
\frac{\cot \left( \frac{\pi}{2} - \theta \right)}{\sec(-\theta) - (\sin^2 \theta + \cos^2 \theta)} & = \frac{\frac{\cos \theta}{\sin \theta}}{\frac{1}{\cos \theta} - 1} = \frac{\cos \theta}{\sin \theta} \cdot \frac{\cos \theta - 1}{1 - \cos \theta} \\
& = \frac{\sin \theta}{1 - \cos \theta} \cdot \frac{1 + \cos \theta}{1 - \cos \theta} = \frac{\sin \theta (1 + \cos \theta)}{1 - \cos^2 \theta} = \frac{\sin \theta (1 + \cos \theta)}{\sin^2 \theta + \cos^2 \theta} = \frac{\sin \theta (1 + \cos \theta)}{\sin^2 \theta} = \frac{\sin \theta (1 + \cos \theta)}{\sin \theta} = \sec \theta + \cot \theta
\end{align*}
\]

\[ \frac{\sin \left( \frac{\pi}{2} - \theta \right)}{1 - \sin(-\theta)} + \cot \left( \frac{\pi}{2} - \theta \right) = \sec \theta \]

\[
\begin{align*}
\frac{\sin \left( \frac{\pi}{2} - \theta \right)}{1 - \sin(-\theta)} + \cot \left( \frac{\pi}{2} - \theta \right) & = \frac{\cos \theta}{1 + \sin \theta} + \frac{\cos \theta}{\sin \theta} = \frac{\cos \theta}{1 + \sin \theta} + \frac{\sin \theta (1 + \sin \theta)}{(1 + \sin \theta) \cos \theta} = \frac{\cos \theta + \sin \theta + \sin^2 \theta}{(1 + \sin \theta) \cos \theta} = \frac{1 + \sin \theta}{(1 + \sin \theta) \cos \theta} = \frac{1}{\cos \theta} = \sec \theta
\end{align*}
\]

\[ \text{Note: you do not need to know } \cos(-\theta) = \cos \theta \quad \text{and} \quad \sin(-\theta) = -\sin \theta \text{ for the exam} \]
5. Below is the graph of a function of the form $f(x) = A\cos(\omega x - \phi) + B$.

- What are the exact values of $A$, $\omega$, $\phi$ and $B$ for the model equation? 
  $[A = 3, \ \omega = \pi, \ \phi = 0, \ B = 2]$

- Find the amplitude, phase shift, vertical shift, period and range of this equation.
  $A = 3, \ \text{ver. shift} = 2, \ \text{range} = [-1, 5], \ \text{per} = 2$

6. The simple harmonic motion of an object is given by $d = -20 \sin \left(\frac{\pi t}{4}\right)$ where $d$ is in cm and $t$ is in seconds.

- What is the maximum displacement of the object? (Don’t forget correct units!) 
  $20 \text{ cm}$

- What is the frequency of the object? (Don’t forget correct units!)
  $\text{freq.} = \frac{1}{\text{per}} \quad \text{per} = \frac{2\pi}{\frac{\pi}{4}} = 2 \pi \cdot \frac{4}{\pi} = 8 \Rightarrow \text{freq.} = \frac{1}{8} \text{ oscillations per sec.}$

- What is the time required for one oscillation? (Don’t forget correct units!)
  $\text{per} = 8 \text{ sec}$

- Describe the initial movement of the object i.e. does the object start from rest moving upward or downward or is the object pulled up or down and then released?

  The object starts from rest moving downward.
7. Find an equation for the graph below.

Graph looks like \( \tan x \)

- \( B = 2 \)
- \( A = 2 \)
- \( \text{period} = \frac{2\pi}{3} = \frac{2\pi}{w} \)
  \( \Rightarrow \) \( w = 3 \)
- \( \text{p.s.} = 0 = \phi \)

\[ y = 2 + \tan (3x) + 2 \]

8. Evaluate the following expressions.

- \( \cos^{-1} \left( \cos \left( \frac{13\pi}{10} \right) \right) = \)
  \[ \cos \left( \frac{13\pi}{10} \right) < 0 \] So \( \cos^{-1} (\cos \left( \frac{13\pi}{10} \right)) \) is in quadrant II
  \[ \cos^{-1} (\cos \left( \frac{13\pi}{10} \right)) = \frac{7\pi}{10} \]

- \( \tan (\cos^{-1} (1)) = \)
  \[ \cos^{-1} (1) = 0 \] \( \tan (0) = 0 \)
9. Find the **exact** value of the following expressions. Be sure to show your work, including a reference triangle, trig identity, or unit circle if appropriate.

\[ \sin^{-1} \left( -\frac{\sqrt{3}}{2} \right) = \frac{\pi}{3} \]

\[ \csc \left( \cos^{-1} \left( -\frac{1}{5} \right) \right) = \sqrt{24} \]

\[ 1 + b^2 = 25 \]
\[ b^2 = 24 \Rightarrow b = \sqrt{24} \]

\[ \tan \left( \frac{\pi}{6} + \sin^{-1} \left( -\frac{24}{25} \right) \right) = \frac{1 + \tan \left( \frac{\pi}{6} \right) \cdot \tan \theta}{1 - \tan \left( \frac{\pi}{6} \right) \cdot \tan \theta} \]

\[ a^2 + 5 + 6 = 625 \]
\[ a^2 = 49 \]
\[ a = 7 \]

\[ \sin \left( \frac{7\pi}{8} \right) \cos \left( \frac{\pi}{8} \right) + \cos \left( \frac{7\pi}{8} \right) \sin \left( \frac{\pi}{8} \right) = \sin \left( \frac{7\pi}{8} + \frac{\pi}{8} \right) = \sin \left( \frac{3\pi}{4} \right) = \frac{\sqrt{2}}{2} \]

\[ \cos \left( \frac{13\pi}{12} \right) = \cos \left( \frac{4\pi}{12} + \frac{9\pi}{12} \right) = \cos \left( \frac{\pi}{3} + \frac{3\pi}{4} \right) = \cos \left( \frac{\pi}{3} \right) \cos \left( \frac{3\pi}{4} \right) - \sin \left( \frac{\pi}{3} \right) \sin \left( \frac{3\pi}{4} \right) \]
\[ = \frac{\sqrt{3}}{2} \cdot \frac{-\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{-1}{2} = \frac{-1 - \sqrt{3}}{2\sqrt{2}} \]
10. Give \( \csc A = \frac{61}{60}, \ \frac{5\pi}{2} < A < 3\pi, \ \sec B = \frac{-7}{4} \) and \( \sin B < 0 \), find the exact value of each of the following.

- \( \sin \left( \frac{A}{2} \right) = \frac{A}{2} \) is in quadrant III, so \( \sin \left( \frac{A}{2} \right) < 0 \)
  \[
  \sin \left( \frac{A}{2} \right) = -\sqrt{\frac{1-\cos A}{2}} = -\sqrt{\frac{1-\left(-\frac{11}{60}\right)}{2}} = -\sqrt{\frac{1+\frac{11}{60}}{2}}
  \]

- \( \cos(2B) = 2\cos^2 B - 1 = 2 \left( \frac{16}{49} \right) - 1 = \frac{32}{49} - 1 = \frac{-17}{49} \)
  \[
  \cos B = -\frac{4}{7}
  \]

- \( \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} \)
  \[
  \tan A = \frac{60}{11}, \ \tan B = \frac{\sqrt{3}}{4}
  \]
  \[
  \sin B < 0 \Rightarrow B \in \text{quadrant III or IV}
  \]

- \( \cot \left( \frac{B}{2} \right) = \frac{1}{\tan \left( \frac{B}{2} \right)} \)
  \[
  B \text{ is in quadrant III}
  \]

11. Verify the equation \( \cos(2x) = \frac{\sec x \csc x - 2 \tan x}{\tan x + \cot x} \).

\[
\frac{\sec x \cdot \csc x - 2 \tan x}{\tan x + \cot x} = \frac{\left( \frac{1}{\cos x} \cdot \frac{1}{\sin x} - 2 \cdot \frac{\sin x}{\cos x} \right)}{\left( \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right)}
\]

\[
= \left( \frac{1}{\sin x \cdot \cos x} - \frac{2 \sin^2 x}{\sin x \cdot \cos x} \right) = \frac{1}{\sin^2 x + \cos^2 x}
\]

\[
= 1 - 2 \sin^2 x = 1 - 2 \sin^2 x = \cos(2x)
\]