Topics You Should Be Comfortable With

• Can you work with select mortality models? By this, I mean:
  – (a) Can you find the probability of survival \( t_p[x] \) given \( \mu[x+s] \)?
  – (b) Can you fill in the columns of a select life table given non-select life table information?
  – (c) Can you use either (a) or (b) above to find the value of benefits and annuities?

• Can you find the value of insurance premiums under various benefit and funding schemes? With, and Without, Expenses?

• Can you integrate? By parts? Substitution? Constant Mortality Models?

In the 6 questions below, we explore many of these topics. Some more practice can be found in our textbook (2nd edition:)

• **Select Models and Benefits/Annuities:** Ex. 3.6, 3.7, 3.8, 4.16, 5.8.

• **Premiums:** Ex. 6.2, 6.3, 6.7, 6.15.
1. A select group has $\mu_{[x]+s} = \frac{1}{2} \cdot \mu_{x+s}$, for all $s \geq 0$. It follows that

(a) $tp_{[x]} = 0.5 \cdot tp_x$
(b) $tp_{[x]} = (tp_x)^2$
(c) $tp_{[x]} = \sqrt{tp_x}$
(d) $tp_{[x]} = 2 \cdot tp_x$
(e) None of the above

Answer: (c)

\[
tp_{[x]} = e^{-\int_0^t \mu_{[x]+s} ds} = e^{-\frac{1}{2} \int_0^t \mu_{x+s} ds} = (e^{-\int_0^t \mu_{x+s} ds})^{\frac{1}{2}} = \sqrt{tp_x}
\]
2. For a 2-year select mortality model, you are given:

\[ q_{x+1} = 0.90 \times q_{x+1} \]
\[ l_{76} = 98000 \]
\[ l_{77} = 96000 \]  

(2)

Solve for \( l_{75}+1 \).

(a) 98691.6
(b) 97796.3
(c) 97000.0
(d) 96550.0
(e) None of the above

ANSWER: (b)

In general, if \( q_{x+1} = \alpha \times q_{x+1} \), then

\[
1 - \frac{l_{x+2}}{l_{x+1}} = \alpha \left( 1 - \frac{l_{x+2}}{l_{x+1}} \right) \\
\Rightarrow l_{x+1} = \frac{l_{x+2}}{(1 - \alpha) + \alpha \frac{l_{x+2}}{l_{x+1}}} \]  

(3)

So, for our case, with \( \alpha = 0.9 \), it follows that

\[
l_{x+1} = \frac{96000}{0.1 + 0.9 \times \frac{96000}{98000}} = 97796.2577963 \approx 97796.3. \]  

(4)
3. Conditioned on knowledge of the constant force of interest $\delta$ and constant force of mortality $\mu$, the expected present value of a benefit $Z$ is

$$E[Z \mid \mu, \delta] = \frac{e - 1}{\mu + \delta}.$$  \hspace{1cm} (5)

Here, interest is compounded continuously and $T_x$ is distributed continuously. If the constant $\delta + \mu$ is not known exactly, but instead uniformly distributed on $[1, e]$, then if this benefit is funded by a single premium $P$, we know

(a) $P = 1$
(b) $P = e$
(c) $P = 3.5$
(d) $P = 3.963$
(e) $P = $ None of the above

ANSWER: (a)

$$P = E[Z] = E[E[Z \mid \mu, \delta]]$$
$$= \int_1^e \frac{e - 1}{x} \cdot \frac{1}{e - 1} \, dx = \ln(e) - \ln(1)$$ \hspace{1cm} (6)
$$= 1 - 0 = 1.$$
4. Consider an insurance contract issued to \((x)\) where the Equivalence Pricing Principle holds and where (i) mortality follows an exponential distribution with parameter \(\lambda > 0\) and (ii) the constant force of interest \(\delta > 0\) is given. The contract provides that a continuous, non-level benefit \(b_T = e^{\lambda T_x}\) is paid upon time of death. If this contract is funded by a single premium \(P\), then \(P =\)

(a) \(\lambda \delta\)
(b) \(\lambda + \frac{\lambda^2}{\delta}\)
(c) \(\frac{\delta}{\lambda}\)
(d) \(\frac{\lambda}{\delta}\)
(e) None of the above

Answer: (d)

We compute \((Y, Z) = (1, b_T \cdot v^T_x) = (1, e^{(\lambda - \delta)T_x})\) and so

\[
P = \frac{E[Z]}{E[1]} = E[Z] = \int_0^\infty e^{(\lambda - \delta)t} \cdot \lambda e^{-\lambda t} dt = \int_0^\infty \lambda e^{-\delta t} dt = \frac{\lambda}{\delta}
\] (7)
5. For a fully discrete whole life insurance of 1000 on \((x)\), you are given:

- The annual per policy expense is 1.
- There is an additional first year expense of 20.
- The claim settlement expense of 50 is payable when the claim is paid.
- All expenses, except the claim settlement expense, are paid at the beginning of the year.
- \(d = \frac{1}{20} = 0.05\)
- \(A_x = \frac{36}{143} \approx 0.25175\)

Calculate the level expense-loaded premium using the equivalence principle.

(a) \(P = 15\)
(b) \(P = 20\)
(c) \(P = 25\)
(d) \(P = 25.78\)
(e) \(P = \text{None of the above}\)

**ANSWER:** (b)

\[
P \ddot{a}_x = 1000A_x + \ddot{a}_x + 20 + 50A_x = 20 + \ddot{a}_x + 1050A_x
\]

\[
\Rightarrow P = \frac{1050A_x + \ddot{a}_x + 20}{\ddot{a}_x}
\]

\[
= 1 + 1050 \frac{A_x}{1 - A_x} + \frac{20d}{1 - A_x}
\]

\[
= 1 + \frac{1050}{20} \frac{A_x}{1 - A_x} + \frac{20 \cdot 1/20}{1 - A_x}
\]

\[
= 1 + \frac{1 + 52.5A_x}{1 - A_x}
\]

\[
= 1 + \frac{1 + 105 \frac{36}{143}}{1 - \frac{36}{143}}
\]

\[
= 1 + 19 = 20.
\]
6. For a special fully discrete 5-year deferred whole life insurance of 100,000 on (40), you are given:

- The death benefit during the 5-year deferral period is return of benefit premiums paid without interest.
- Annual benefit premiums are payable only during the deferral period.
- \( i = 0.06 \)
- \( (IA)_{40:5}^1 = 0.04042 \)
- \( v^5 \cdot 5p_{40} = 0.73529 \)
- \( A_{45} = 0.20120 \)
- \( \ddot{a}_{40} = 14.8166 \)
- \( \ddot{a}_{45} = 14.1121 \)

Calculate the annual benefit premiums \( P \).

(a) \( P = 2000 \)
(b) \( P = 3000 \)
(c) \( P = 3363 \)
(d) \( P = 3963 \)
(e) None of the above

**ANSWER:** (c)

Our EPP balance equation is

\[
P\ddot{a}_{40:5} = P \cdot (IA)_{40:5}^1 + 100000v^5 \cdot 5p_{40}A_{45}.
\] (9)

It follows that

\[
P = \frac{100000v^5 \cdot 5p_{40}A_{45}}{\ddot{a}_{40:5} - (IA)_{40:5}^1} = \frac{100000 \cdot v^5 \cdot 5p_{40} \cdot A_{45}}{\ddot{a}_{40} - v^5 \cdot 5p_{40} \cdot A_{45} - (IA)_{40:5}^1} = \frac{100000 \cdot 0.73529 \cdot 0.20120}{14.8166 - 0.73529 \cdot 14.1121 - 0.04042} = 3362.514491.
\] (10)