Basic Information:

\[ V^C(S, t) = e^{-r(T-t)} \mathbb{E} [(S_T - K)_+ | S_t = S] \]
\[ = Se^{-\delta(T-t)} N(d_1) - Ke^{-r(T-t)} N(d_2) \]

\[ V^P(S, t) = e^{-r(T-t)} \mathbb{E} [(K - ST)_+ | S_t = S] \]
\[ = Ke^{-r(T-t)} N(-d_2) - Se^{-\delta(T-t)} N(-d_1) \]

\[ d_1 = \frac{\ln \left( \frac{S}{K} \right) + (r - \delta + \frac{1}{2} \sigma^2)(T - t)}{\sigma \sqrt{T - t}} \]
\[ d_2 = d_1 - \sigma \sqrt{T - t} \]

\[ N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{z^2}{2}} dz. \]

1. Consider the continuous Black Scholes environment with \( \delta = 0 \) and initial stock value \( S_0 = K \). What happens to the initial \( \Delta^C(S, 0) \) of a European Call when \( T \to \infty \)? (5 points.)

**Answer:**

As \( T \to \infty \), we see that the value of the option becomes indistinguishable from the value of the stock. It makes sense that the Delta of the Call Option should approach that of a stock:

\[ \lim_{T \to \infty} \Delta^C(S, 0) = \lim_{T \to \infty} N(d_1) \]
\[ = \lim_{T \to \infty} N \left( \frac{0 + (r - 0 + \frac{1}{2} \sigma^2)(T)}{\sigma \sqrt{T}} \right) \]
\[ = 1. \]
2. Consider a European call option and a European put option on a nondividend-paying stock. You are given:

- The current price of the stock is 60.
- The call option currently sells for 0.15 more than the put option.
- Both the call option and put option will expire in 4 years.
- Both the call option and put option have a strike price of 70.

Calculate

- the continuously compounded risk-free interest rate. (5 points.)
- the initial value of Delta on the strategy of owning one call option and selling one put option. (5 points.)

Answer:

For the continuous rate, we use Put-Call Parity to discover

\[ 0.15 = V^C(60, 0) - V^P(60, 0) = 60 - 70e^{-4r} \]

\[ \Rightarrow r = 0.0391625. \]  \hspace{1cm} (3)

For the delta of the strategy, it follows that

\[ \Delta^{Strategy}(S, 0) = \Delta^C(S, 0) - \Delta^P(S, 0) = \frac{\partial}{\partial S}(S - Ke^{-rT}) \]

\[ \therefore \Delta^{Strategy}(S, 0) = 1. \]  \hspace{1cm} (4)