

1. Let

$$A = \begin{pmatrix} 1 & -3 & -5 \\ 1 & 1 & -2 \\ 1 & -3 & 1 \\ 1 & 1 & 4 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} -6 \\ 1 \\ 1 \\ 6 \end{pmatrix}.$$

If the Gram-Schmidt process is applied to determine an orthonormal basis for $C(A)$ and a QR factorization of A , then, after the first two orthonormal vectors \vec{q}_1 and \vec{q}_2 are computed, we have

$$Q = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & - \\ \frac{1}{2} & \frac{1}{2} & - \\ \frac{1}{2} & -\frac{1}{2} & - \\ \frac{1}{2} & \frac{1}{2} & - \end{pmatrix}, \quad R = \begin{pmatrix} 2 & -2 & - \\ 0 & 4 & - \\ 0 & 0 & - \end{pmatrix}.$$

(a) Finish the process. Determine \vec{q}_3 and fill in the third columns of Q and R . [6 points]

- (b) Use the QR factorization to find the least squares solution of $A\vec{x} = \vec{b}$. [4 points]