

1. Let

$$A = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 4 & -3 & 0 \\ 1 & 2 & 1 & 5 \end{bmatrix}.$$

- (a) Use elimination to find the reduced row echelon matrix R . Identify the pivot columns and free columns. *[3 points]*
- (b) How many special solutions does $A\vec{x} = \vec{0}$ have? Write the nullspace $N(A)$ as the span of the special solution(s) *[2 points]*
- (c) Write the column space, $C(A)$, of A as the span of two vectors. *[1 point]*

2. Suppose an m by n matrix has r pivots. Answer the following questions: *[1 point each]*
- (a) The number of special solutions is _____.
 - (b) The nullspace contains only $\vec{x} = \vec{0}$ when $r =$ _____.
 - (c) The column space is all of \mathbb{R}^m when $r =$ _____.
3. Construct a matrix A such that its nullspace contains all multiples of $(2, -1, 3, 1)$. *[1 point]*.