

1. Answer the following questions about the matrices below:

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -4 & 0 & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 11 & -5 & 3 \\ 1 & 3 & -1 & 2 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{pmatrix}.$$

(a) Compute BC . What effect does B have on the rows of C ? [1 point]

(b) Compute ABC . What effect does A have on the rows of BC ? [1 point]

(c) Write the inverse matrix, A^{-1} , which reverses the effect of A on matrix rows. [1 point]

2. Write down the augmented matrix $[A \mid \vec{b}]$ for the following system of equations. Use elimination to reduce the system to upper triangular form, and then back substitute for z, y, x . Show all your steps and write down the elimination (row exchange) matrix used in each step. [4 points]

$$\begin{aligned}x + 2y + z &= 1 \\3x + 7y + 3z &= 1 \\-2x - 3y - 4z &= -1\end{aligned}$$

3. Choose the numbers p, q, r, s in this augmented matrix so that there is (a) no solution (b) infinitely many solutions.

$$(A | \vec{b}) = \left(\begin{array}{ccc|c} 3 & 12 & -6 & p \\ 0 & 1 & 3 & q \\ 0 & 0 & s & r \end{array} \right)$$

Which of the numbers p, q, r or s have no effect on the solvability? [3 points]