NAME: SOLUTIONS **Quiz 3**, MTH314-001

1. Answer the following questions about the matrices below:

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -4 & 0 & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 11 & -5 & 3 \\ 1 & 3 & -1 & 2 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{pmatrix}.$$

(a) Compute BC. What effect does B have on the rows of C? *[2 points]*

$$BC = \begin{pmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{pmatrix}$$

B is a row exchange matrix. It exchanges rows 1 and 2 of matrix C.

(b) Compute ABC. What effect does A have on the rows of BC? *[2 points]*

$$ABC = \begin{pmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 0 & -11 & 5 & -3 \end{pmatrix}$$

A is an elimination matrix. It subtracts 4 times row 1 from row 4 of BC.

(c) Write the inverse matrix, A^{-1}, which reverses the effect of A on matrix rows. *[1 point]*

$$A^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 4 & 0 & 0 & 1 \end{pmatrix}$$

This matrix adds 4 times row 1 to row 4.

2. Write down the augmented matrix $[\, A \mid \vec{b}\,]$ for the following system of equations. Use elimination to reduce the system to upper triangular form, and then back substitute for z, y, x. Show all your steps and write down the elimination (row exchange) matrix used in each step. *[4 points]*

$$\begin{aligned} x + 2y + z &= 1 \\ 3x + 7y + 3z &= 1 \\ -2x - 3y - 4z &= -1 \end{aligned}$$

Augmented Matrix $[A \mid \vec{b}] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 3 & 7 & 3 & 1 \\ -2 & -3 & -4 & -1 \end{array}\right]$

Let's perform elimination

$E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad E_{21}[A \mid \vec{b}] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 0 & -2 \\ -2 & -3 & -4 & -1 \end{array}\right]$

(row 2 − 3 row 1)

$E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ $\quad E_{31}E_{21}[A \mid \vec{b}] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 1 & -2 & 1 \end{array}\right]$

(row 3 + 2 row 1)

$E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$ $\quad E_{32}E_{31}E_{21}[A \mid \vec{b}] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & -2 & 3 \end{array}\right]$

(row 3 − row 2)

We have now reached upper triangular form.

Back-Substituting, we have $\boxed{z = -\frac{3}{2}}$, $\boxed{y = -2}$

and $x = 1 - z - 2y = 1 + \frac{3}{2} + 4 = \frac{13}{2}$

$\boxed{x = \frac{13}{2}}$

3. Choose the numbers p, q, r, s in this augmented matrix so that there is (a) no solution (b) infinitely many solutions. *[3 points]*

$$\left(A \mid \vec{b}\right) = \left(\begin{array}{ccc|c} 3 & 12 & -6 & p \\ 0 & 1 & 3 & q \\ 0 & 0 & s & r \end{array}\right)$$

Which of the numbers p, q, r or s have no effect on the solvability?

(a) There is no solution when $s=0$ and $r \neq 0$

Why? the last row reads $0z = r$. When $r \neq 0$, there is no solution.

(b) There are infinitely many solutions when $s=0$ and $r=0$.

Why? the last row now reads $0z=0$ which permits an infinite number of solutions.

p and q have no effect on the solvability of the problem