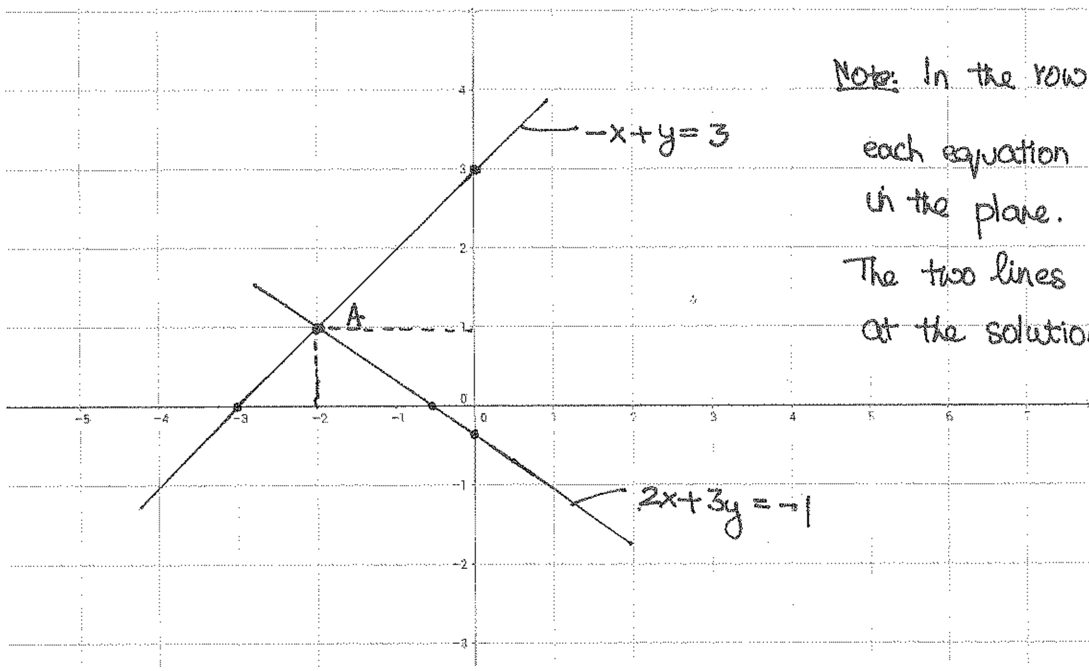
NAME: Solutions **Quiz 2**, MTH314-001

1. Consider the following set of equations:

$$2x + 3y = -1$$
$$-x + y = 3.$$

$\xrightarrow[\text{equation}]{\text{Matrix}}$ $\begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

(a) Draw the row picture corresponding to this set of equations. *[1 point]*



Note: In the row picture, each equation is a line in the plane. The two lines intersect at the solution

(b) Using part (a), write $(-1, 3)$ as a linear combination of $(2, -1)$ and $(3, 1)$. *[1 point]*

The two lines intersect at the point $(-2, 1)$ which is the solution to the given set of equations.

Since $(2,-1)$, $(3,1)$ are the columns of the coefficient matrix and $(-1,3)$ is the right hand side vector $\vec{b}$ we have $\boxed{-2\begin{pmatrix} 2 \\ -1 \end{pmatrix} + 1\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}}$

2. Find a combination $a\,\vec{v}_1 + b\,\vec{v}_2 + c\,\vec{v}_3$ that gives the zero vector: *[2 points]*

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} \qquad \vec{v}_3 = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}$$

$$1\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + 1\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} - 1\begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Hence $\boxed{\begin{aligned} a &= 1 \\ b &= 1 \\ c &= -1 \end{aligned}}$

(Circle the correct choices)

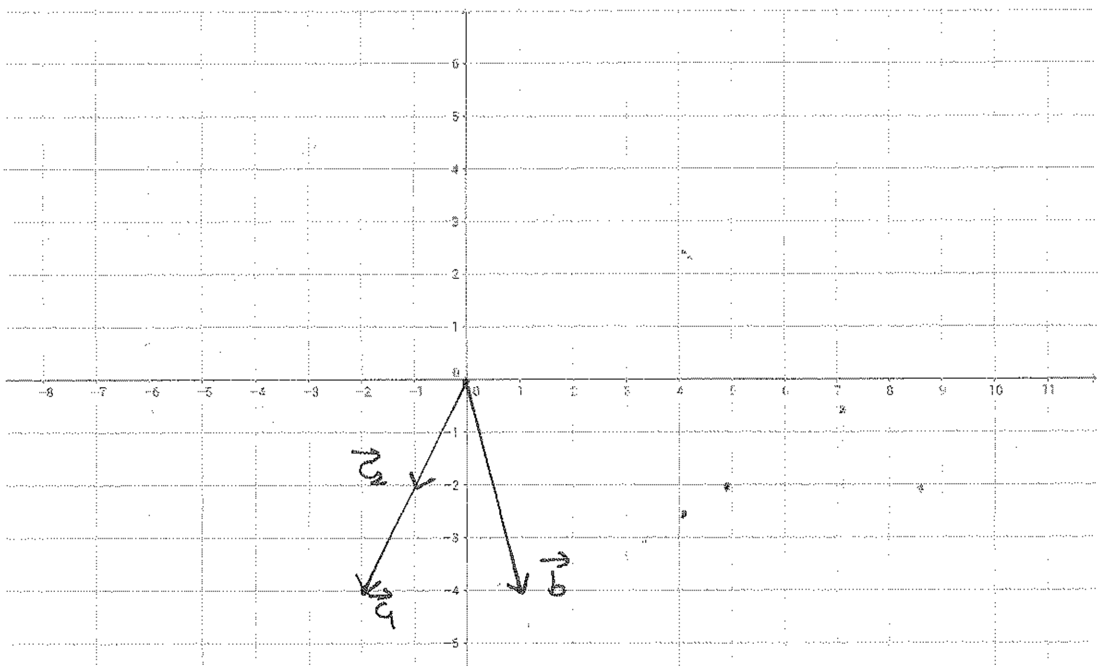
These vectors are: independent dependent

The three vectors lie on a: line plane 3D space

Note: this is just one of the possible combinations

3. Consider the two equations $-y = 2x + 1$ and $-2y = 4x - 4$.

   (a) Draw the column picture corresponding to this set of equations. *[1 point]*



$-y = 2x + 1$
$-2y = 4x - 4$

Can be written as a matrix-vector equation

$$\begin{bmatrix} -2 & -1 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} -2 \\ -4 \end{bmatrix}}_{\vec{c}_1} \quad \underbrace{\begin{bmatrix} -1 \\ -2 \end{bmatrix}}_{\vec{c}_2} \quad \underbrace{\begin{bmatrix} 1 \\ -4 \end{bmatrix}}_{\vec{b}}$$

   (b) Using your graph from part (a), explain why the following matrix–vector equation does not have a solution. *[1 point]*

$$\begin{pmatrix} -2 & -1 \\ -4 & -2 \end{pmatrix} \vec{x} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}.$$

The columns of the coefficient matrix are plotted as vectors $\vec{c}_1$ and $\vec{c}_2$ in the graph. We see they lie on a line. All linear combinations of $\vec{c}_1$ and $\vec{c}_2$ also lie on the same line. However, the right hand side vector – plotted as $\vec{b}$ on the graph – does not lie on this line. Hence the matrix-vector equation does not have a solution.

4. Which number p makes this system singular and which right hand side q gives infinitely many solutions? Find the solution that has $x_3 = 1$. **Show all steps used to arrive at your answer.** *[4 points]*

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 2 \\ 2x_1 + 6x_2 + x_3 &= 7 \\ 2x_2 + px_3 &= q. \end{aligned}$$

Subtracting 2x row ① from row ② we get

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 2 \\ 2x_2 + 7x_3 &= 3 \\ 2x_2 + px_3 &= q \end{aligned}$$

If $\boxed{p=7}$ performing another step of elimination results in

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 2 \\ 2x_2 + 7x_3 &= 3 \\ \underline{0}\, x_3 &= \underline{q-3} \end{aligned}$$

Since 0 cannot be a pivot, this system is singular for $p=7$.

If $\boxed{q=3}$ then the last equation becomes $0x_3 = 0$ which permits infinitely many solutions.

Now if $x_3 = 1$, back substitution gives

$2x_2 + 7 = 3$ or $\boxed{x_2 = -2}$

$x_1 - 4 - 3 = 2$ or $\boxed{x_1 = 9}$

Hence (one of the) solutions is $\boxed{x_1 = 9,\ x_2 = -2,\ x_3 = 1}$