NAME: _____ **Quiz 1**, MTH314-001

1. For any *unit* vectors $\vec{a}$ and $\vec{b}$, find the dot product of $(\vec{a} + \sqrt{3}\,\vec{b})$ and $(\vec{a} - \sqrt{3}\,\vec{b})$. Show all your work. *[2 points]*

$$(\vec{a} + \sqrt{3}\,\vec{b}) \cdot (\vec{a} - \sqrt{3}\,\vec{b}) = \vec{a}\cdot\vec{a} + \sqrt{3}\,\vec{b}\cdot\vec{a} - \sqrt{3}\,\vec{a}\cdot\vec{b} - 3\,\vec{b}\cdot\vec{b}$$

$$= \vec{a}\cdot\vec{a} - 3\,\vec{b}\cdot\vec{b} \quad (\text{since } \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a})$$

Note that $\vec{a}\cdot\vec{a} = \|\vec{a}\|\|\vec{a}\|\cos 0 = (1)(1)\cdot 1 = 1$ (since $\|\vec{a}\| = 1$; Unit vector)

Similarly $\vec{b}\cdot\vec{b} = 1$

Hence $(\vec{a} + \sqrt{3}\,\vec{b}) \cdot (\vec{a} - \sqrt{3}\,\vec{b}) = 1 - 3 = -2.$

2. For each of the following, answer **True** or **False**. Provide a brief justification for your answer. *[2 points each]*.

(a) If $\vec{p}$ is perpendicular to $\vec{q}$ and $\vec{r}$, then $\vec{p}$ is perpendicular to $5\,\vec{q} - \frac{1}{2}\,\vec{r}$.

TRUE

$$\vec{p}\cdot(5\vec{q} - \tfrac{1}{2}\vec{r}) = 5\,\vec{p}\cdot\vec{q} - \tfrac{1}{2}\,\vec{p}\cdot\vec{r}$$

$$= 0 \quad \text{since } \vec{p}\cdot\vec{q} = 0 \ (\vec{p}, \vec{q} \text{ perpendicular})$$

$$\vec{p}\cdot\vec{r} = 0 \ (\vec{p}, \vec{r} \text{ perpendicular})$$

Hence $\vec{p}$ is perpendicular to $(5\,\vec{q} - \frac{1}{2}\vec{r})$

(b) Suppose that $\|\vec{u}\| = 3$ and $\|\vec{v}\| = 2$. The minimum possible value of $\|\vec{u} - \vec{v}\| + \vec{u}\cdot\vec{v}$ is achieved when $\vec{u}$ is perpendicular to $\vec{v}$.

FALSE

minimum value is achieved when $\vec{u}$ and $\vec{v}$ point in opposite directions

Note: when $\vec{u}$ is perpendicular to $\vec{v}$, $\|\vec{u} - \vec{v}\| = \sqrt{13}$, $\vec{u}\cdot\vec{v} = 0$

when $\vec{u}, \vec{v}$ are in opposite directions, $\|\vec{u} - \vec{v}\| = 5$, $\vec{u}\cdot\vec{v} = -6$

3. Find unit vectors $\vec{a}$ and $\vec{b}$ that are perpendicular to $\langle 2, -1, 0\rangle$ and to each other. Show all your work. *[3 points]*

The plane perpendicular to $(2, -1, 0)$ has vectors with components $(a, 2a, b)$ where a, b are real numbers

In this plane, $(1, 2, 0)$ and $(0, 0, 1)$ are perpendicular to each other

Hence $\vec{a} = \dfrac{(1, 2, 0)}{\sqrt{1^2+2^2}} = (1, 2, 0)/\sqrt{5}$

$\vec{b} = \dfrac{(0, 0, 1)}{\sqrt{1^2}} = (0, 0, 1)$

Note: this is not the only possible solution. For eg, here's another

$\vec{a} = (1, 2, 1)/\sqrt{6}$ $\quad \vec{b} = (-1, -2, 5)/\sqrt{30}$

4. Fill in the blank below. Provide a brief justification for your answer. *[1 point]*

All possible linear combinations of $\vec{a} = (1, 1, 1)$, $\vec{b} = (1, 2, -1)$ and $\vec{c} = (0, 1, -2)$ fill

a plane

(your answer should be one of the following: a line, a plane, or three-dimensional space)

since $-\vec{a} + \vec{b} - \vec{c} = \vec{0}$