NAME: SOLUTIONS — Quiz 10, MTH314-001

1. Answer the following questions about $A \in \mathbb{R}^{3\times 3}$ below.

$$A = \begin{pmatrix} 3 & 3 & 3 \\ 6 & 2 & 3 \\ 3 & 2 & 2 \end{pmatrix}.$$

(a) Calculate the determinant of A by reducing it to upper triangular form. *[4 points]*

Performing elimination,

$$\begin{bmatrix} 3 & 3 & 3 \\ 6 & 2 & 3 \\ 3 & 2 & 2 \end{bmatrix} \xrightarrow[\substack{R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1}]{} \begin{bmatrix} 3 & 3 & 3 \\ 0 & -4 & -3 \\ 0 & -1 & -1 \end{bmatrix} \xrightarrow[R_3 \to R_3 - \frac{1}{4}R_2]{} \begin{bmatrix} 3 & 3 & 3 \\ 0 & -4 & -3 \\ 0 & 0 & -\frac{1}{4} \end{bmatrix}$$

We have reached upper triangular form.

$|A| = u_{11}\, u_{22}\, u_{33}$ where u_{ii} denote the diagonal entries

$= (3)\,(-4)\,(-\tfrac{1}{4})$

$|A| = 3.$

(b) Calculate $|3A^{\mathrm{T}}|$. *[2 points]*

- determinant is a linear function of each row separately

  in particular $\begin{vmatrix} ta_{11} & ta_{12} & ta_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = t \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

- also $|A| = |A^{\mathrm{T}}|$

Since there are 3 rows in A, $|3A^{\mathrm{T}}| = 3^3\,|A^{\mathrm{T}}| = 3^3\,|A|$

or $|3A^{\mathrm{T}}| = 81.$

(c) Calculate $|A^{-1}| = \det(A^{-1})$. *[2 points]*

$$|A^{-1}| = \frac{1}{|A|}$$

Hence $|A^{-1}| = \frac{1}{3}$.

(d) Calculate the determinant of matrix below. *[2 points]*

$$A_1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 3 & 3 & 0 & 3 \\ 6 & 2 & 0 & 3 \\ 3 & 2 & 0 & 2 \end{pmatrix}.$$

Using the cofactor formula along row 1,

$\det A_1 = (1)\, C_{13}$ whose cofactor $C_{13} = (-1)^{1+3} \begin{vmatrix} 3 & 3 & 3 \\ 6 & 2 & 3 \\ 3 & 2 & 2 \end{vmatrix}$

$= |A|$

Hence $|A_1| = |A| = 3$.