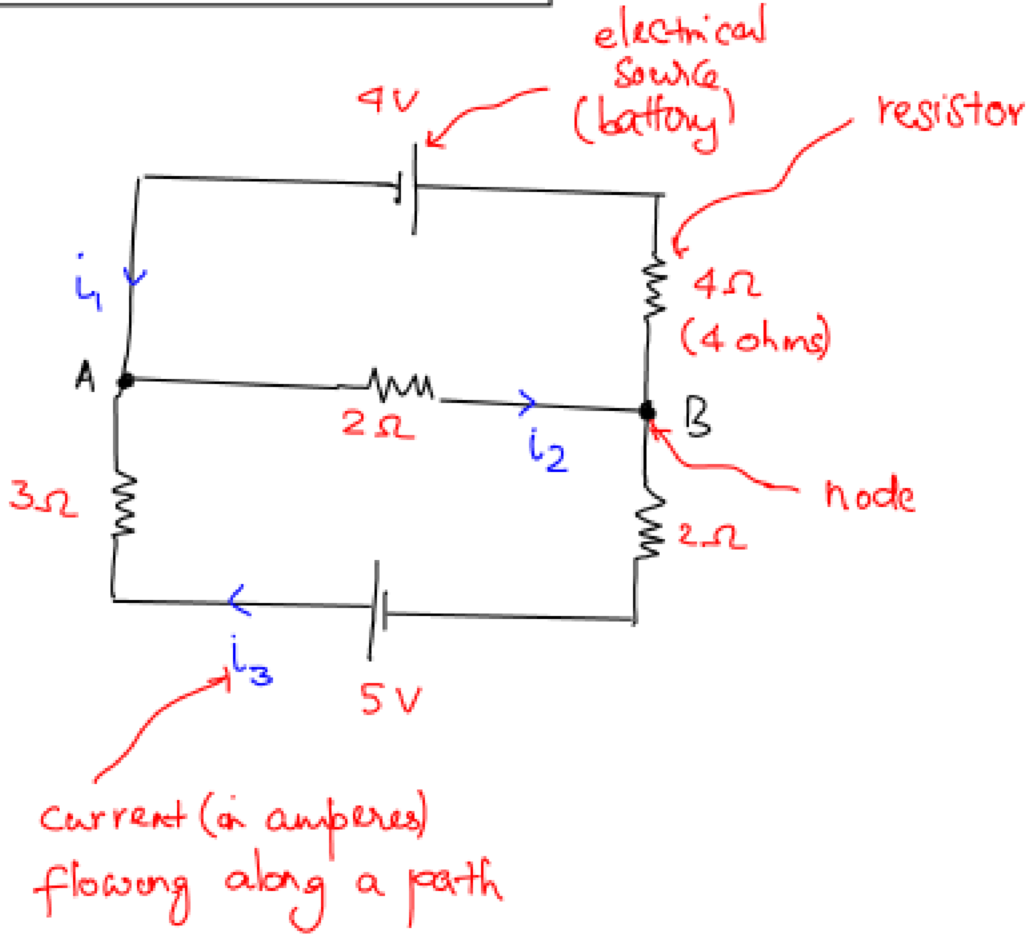
ELECTRICAL NETWORKS



Task: Given the electrical network find the currents i_1, i_2 and i_3

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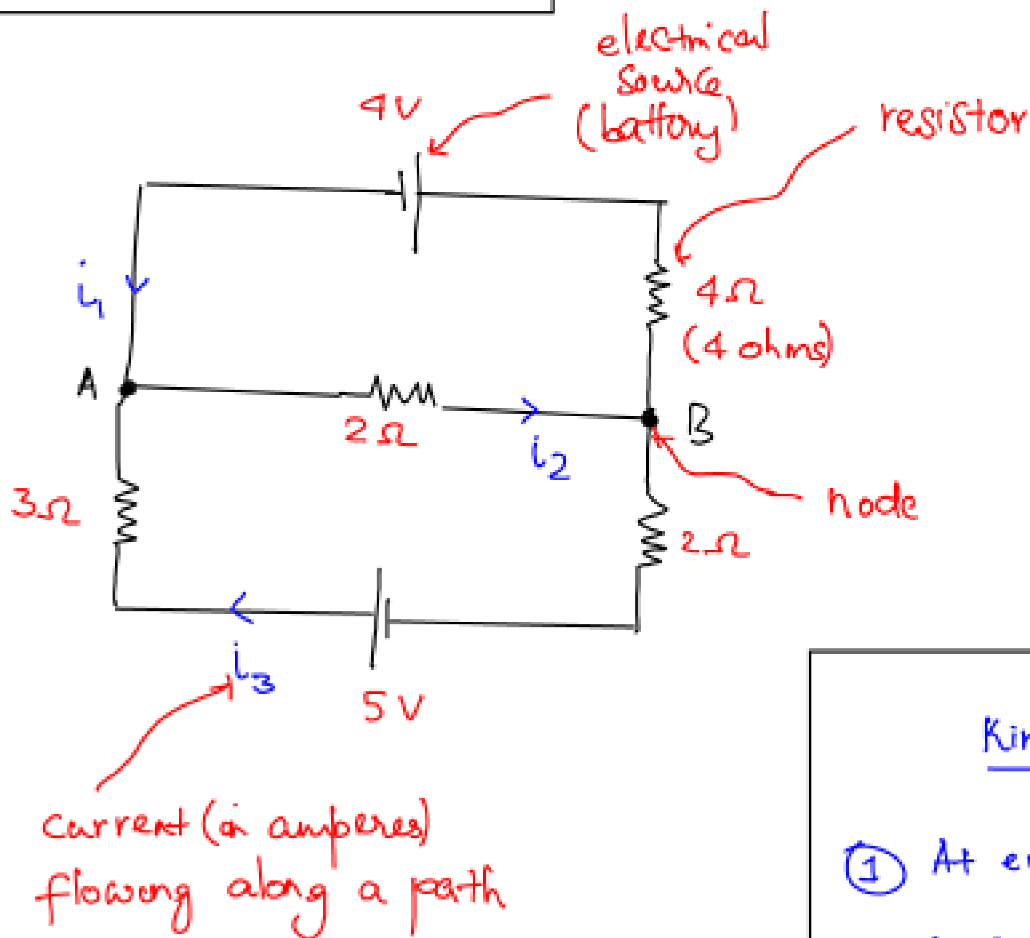


Task: Given the electrical network find the currents i_1, i_2 and i_3

Kirchoff's Laws

1. At every node, the sum of the incoming currents equals the sum of the outgoing currents

2. Around every closed loop, the algebraic sum of the voltage gains must equal the algebraic sum of the voltage drops

Voltage drops are given by <u>Ohm's law</u>

$$E = i R$$

voltage drop (V)

current (A)

resistance (Ω)

Kirchoff's Laws

① At every node, the sum of the incoming currents equals the sum of the outgoing currents

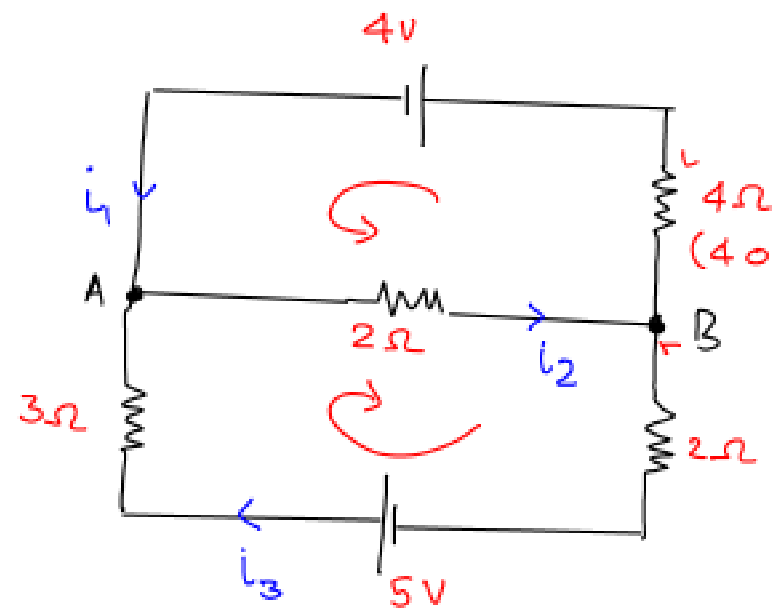
② Around every closed loop, the algebraic sum of the voltage gains must equal the algebraic sum of the voltage drops

(at node A) $\quad i_1 - i_2 + i_3 = 0$

(at node B) $\quad -i_1 + i_2 - i_3 = 0$

(for the top loop) $\quad -4i_1 - 4 - 2i_2 = 0$

(for the bottom loop) $\quad -2i_3 + 5 - 3i_3 - 2i_2 = 0$



Solve using elimination

$$\underbrace{\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 4 & 2 & 0 \\ 0 & 2 & 5 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ -4 \\ 5 \end{bmatrix}}_{\vec{b}}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ -1 & 1 & -1 & 0 \\ 4 & 2 & 0 & -4 \\ 0 & 2 & 5 & 5 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 6 & -4 & -4 \\ 0 & 2 & 5 & 5 \end{array}\right]$$

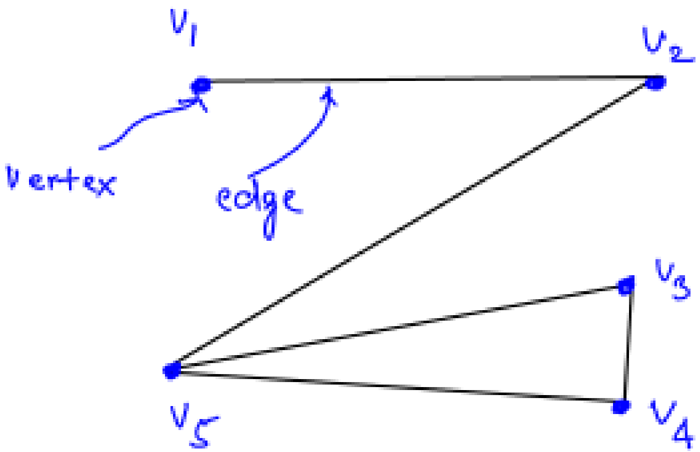
$$\longrightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 2 & 5 & 5 \\ 0 & 6 & -4 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 2 & 5 & 5 \\ 0 & 0 & -19 & -19 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Back substitution yields $i_3 = 1$, $i_2 = 0$, $i_1 = -1$

NETWORKS AND GRAPHS

A graph is defined to be a set of points called <u>vertices</u>, together with a set of (un)ordered pairs of vertices, which are called <u>edges</u>.



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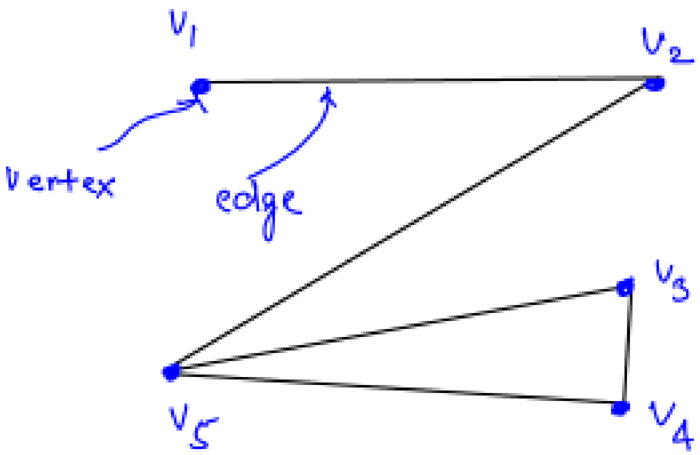

Matrix representation of a graph

(undirected graph)

If a graph contains n vertices, define $A \in \mathbb{R}^{n \times n}$ by

adjacency matrix

$$a_{ij} = \begin{cases} 1 & \text{if } \{v_i, v_j\} \text{ is an edge} \\ 0 & \text{if there is no edge joining } v_i, v_j \end{cases}$$

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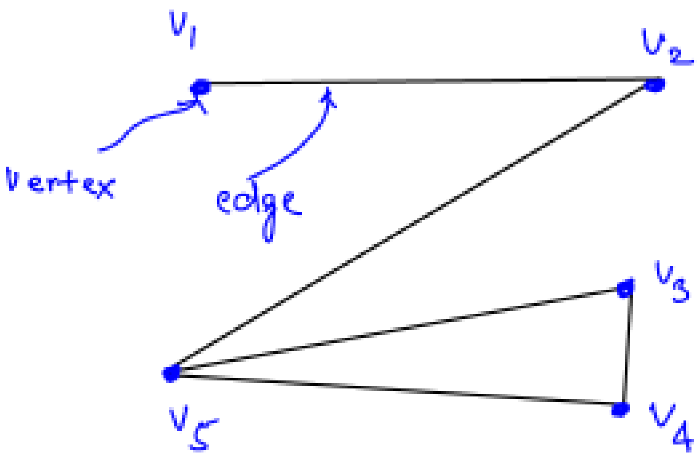

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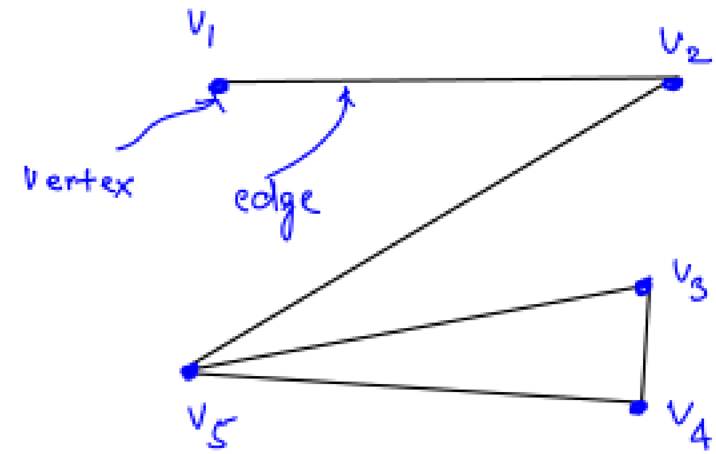
For the above graph,

$$A = \begin{array}{c c} & \begin{array}{ccccc} v_5 & v_2 & v_3 & v_4 & v_5 \end{array} \\ & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} & \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \end{array}$$

A is a symmetric matrix

Walk in a graph

Sequence of edges linking one vertex to another



$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

edges $\{v_1, v_2\}$, $\{v_2, v_5\}$ represent a walk from v_1 to v_5

length of the walk is 2 since it contains 2 edges.

Theorem If $A \in \mathbb{R}^{n \times n}$ is an adjacency matrix of a graph, and $a_{ij}^{(k)}$ represents the (i,j) entry of A^k, then $a_{ij}^{(k)}$ is equal to the number of walks of length k from i to j.

$$A^3 = \begin{bmatrix} 0 & 2 & 1 & 1 & 0 \\ 2 & 0 & 1 & 1 & 4 \\ 1 & 1 & 2 & 3 & 4 \\ 1 & 1 & 3 & 2 & 4 \\ 0 & 4 & 4 & 4 & 2 \end{bmatrix}$$

Example: # of walks of length 3 from v_3 to v_5 $= a_{35}^{(k)} = 4$

STATISTICS - CORRELATION MATRICES

SCORES

STUDENT	ENG	MTH	PHY
S1	61	53	53
S2	63	73	78
S3	78	61	82
S4	65	84	96
S5	63	59	71
AVG	66	66	76

Task: Describe how the three sets of scores are correlated.

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Task: Describe how the three sets of scores are correlated.

(to study how student performance compares between each set of scores)

to allow for any differences in difficulty, adjust the scores to have mean 0.

$$\underset{\sim}{X} = \begin{bmatrix} -5 & -13 & -23 \\ -3 & 7 & 2 \\ 12 & -5 & 6 \\ -1 & 18 & 20 \\ -3 & -7 & -5 \end{bmatrix}$$

the column vectors represent deviations from the mean for each of the three sets of scores

To compare two sets of scores, compute the cosine of the angle between the corresponding column vectors of $\underline{X}$.

$$\underline{X} = \begin{array}{c} \left[\begin{array}{ccc} -5 & -13 & -23 \\ -3 & 7 & 2 \\ 12 & -5 & 6 \\ -1 & 18 & 20 \\ -3 & -7 & -5 \end{array}\right] \\ \begin{array}{ccc} \underbrace{\qquad}_{\vec{x}_1} & \underbrace{\qquad}_{\vec{x}_2} & \underbrace{\qquad}_{\vec{x}_3} \end{array} \end{array}$$

cosine value near 1 $\longrightarrow$ the two sets of scores are highly correlated

$$\cos\theta_{12} = \frac{\vec{x}_1^{\,T}\vec{x}_2}{\|\vec{x}_1\|\,\|\vec{x}_2\|} = -0.038$$

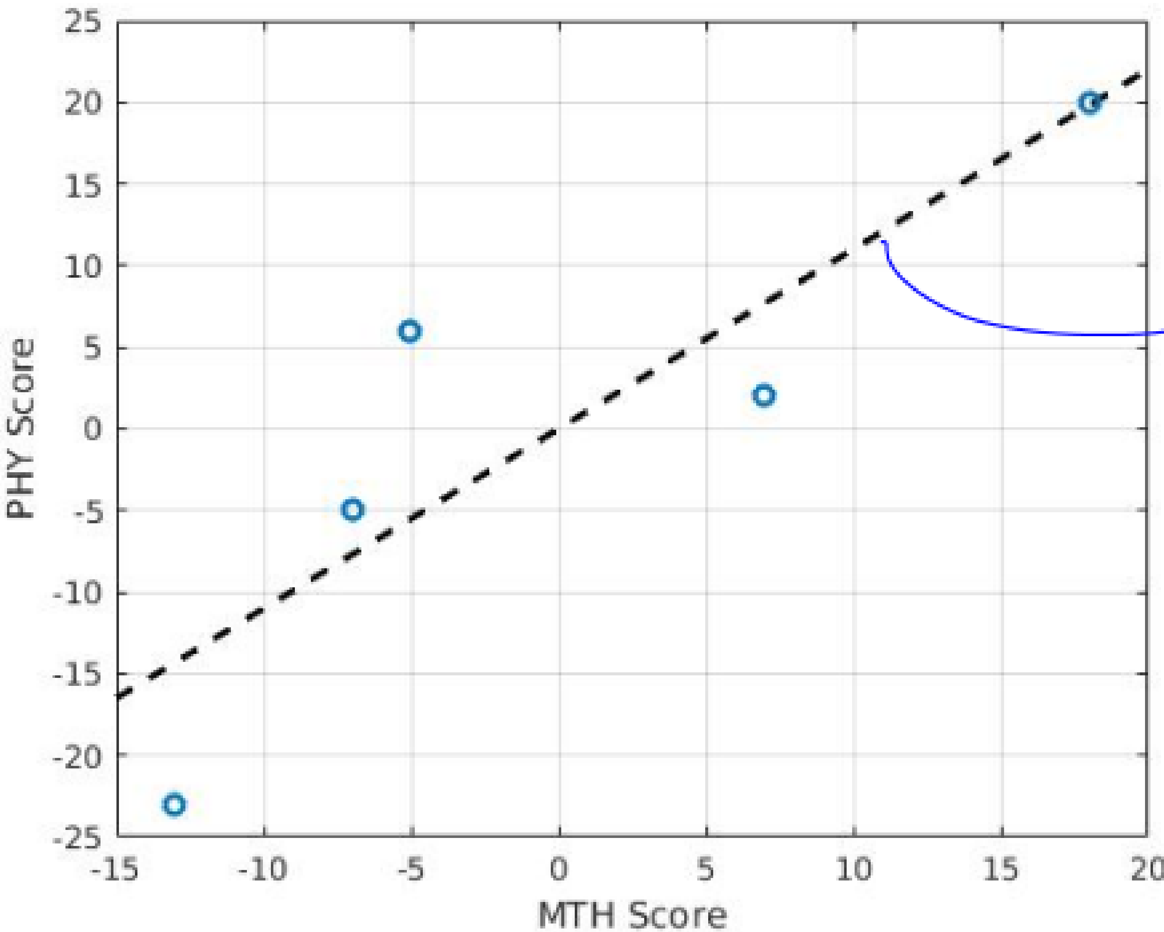
$$\cos\theta_{23} = \frac{\vec{x}_2^{\,T}\vec{x}_3}{\|\vec{x}_2\|\,\|\vec{x}_3\|} = 0.86$$

Note: perfect correlation of 1 means the two sets of translated scores are proportional;

i.e., $\vec{x}_2 = \alpha \vec{x}_1$

the pairs of scores would lie on a line $y = \alpha x$.

$$\underline{X} = \begin{array}{c} \begin{bmatrix} -5 & -13 & -23 \\ -3 & 7 & 2 \\ 12 & -5 & 6 \\ -1 & 18 & 20 \\ -3 & -7 & -5 \end{bmatrix} \\ \begin{array}{ccc} \quad\vec{x}_1 & \quad\vec{x}_2 & \quad\vec{x}_3 \end{array} \end{array}$$



least-squares solution

CORRELATION MATRIX

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$$\begin{array}{ccc} \vec{x}_1 & \vec{x}_2 & \vec{x}_3 \end{array}$$

Scale columns to be unit vectors

$$\vec{u}_1 = \frac{\vec{x}_1}{\|\vec{x}_1\|} \qquad \vec{u}_2 = \frac{\vec{x}_2}{\|\vec{x}_2\|} \qquad \vec{u}_3 = \frac{\vec{x}_3}{\|\vec{x}_3\|}$$

$$U = \begin{bmatrix} -0.36 & -0.52 & -0.73 \\ -0.22 & 0.28 & 0.06 \\ 0.88 & -0.20 & 0.19 \\ -0.07 & 0.73 & 0.63 \\ -0.22 & -0.28 & -0.16 \end{bmatrix}$$

Compute

$$C = U^T U$$

$$C = \begin{bmatrix} 1 & -0.04 & 0.41 \\ -0.04 & 1 & 0.86 \\ 0.41 & 0.86 & 1 \end{bmatrix}$$

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Compute

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Correlation coefficient

$$C = \begin{bmatrix} 1 & -0.04 & 0.41 \\ -0.04 & 1 & 0.86 \\ 0.41 & 0.86 & 1 \end{bmatrix}$$

CORRELATION MATRIX

Correlation Coefficient $= 0$ (uncorrelated)

—"— > 0 (positively correlated)

—"— < 0 (negatively correlated)