## Week 8 - Worksheet - MTH 305 (Spring 2017)

(1) From the graph of $g$, state the intervals on which $g$ is continuous.

(2) Where are each of the following functions discontinuous?
(a) $f(x)=\frac{x^{2}-x-12}{x-4}$
(b) $f(x)=\left\{\begin{array}{lll}\frac{1}{x^{2}} & \text { if } & x \neq 0 \\ 1 & \text { if } & x=0\end{array}\right.$
(c) $f(x)=\left\{\begin{array}{lll}\frac{x^{2}-x-12}{x-4} & \text { if } & x \neq 4 \\ 1 & \text { if } & x=4\end{array}\right.$
(d) $f(x)=\left\{\begin{array}{lll}x+2 & \text { if } & x<0 \\ 2 x^{2} & \text { if } & 0 \leq x \leq 1 \\ 2-x & \text { if } & x>1\end{array}\right.$

At which of these numbers is $f$ continuous from the right, from the left, or neither. Sketch the graph of $f$.
(3) The toll $T$ charged for driving on a certain stretch of a toll road is $\$ 5$ except during rush hours (between 7 AM and 10 AM and between 4 PM and 7 PM ) when the toll is $\$ 7$.
(a) Sketch a graph of $T$ as a function of the time $t$, measured in hours past midnight.
(b) Discuss the discontinuities of this function and their significance to someone who uses the road.
(4) Consider the function

$$
f(x)=\left\{\begin{array}{lll}
\frac{x^{2}+4 x-5}{x-1} & \text { if } & x \neq 1 \\
a & \text { if } & x=1
\end{array}\right.
$$

Find the value of $a$ so that $f$ is continuous everywhere.
(5) Find the values of $A$ and $B$ so that the following function is continuous for all values of $x$

$$
f(x)=\left\{\begin{array}{lll}
A x-B & \text { if } x \leq-1 \\
2 x^{2}+3 A x+B & \text { if }-1<x \leq 1 \\
4 & \text { if } x>1
\end{array}\right.
$$

(6) Use the intermediate value theorem to show that there is a root of the equation $x^{4}+x-3=0$ between 1 and 2 .
(7) Show that the function $f(x)=1-\sqrt{4-x^{2}}$ is continuous on the interval $[-2,2]$.

