Week 6 – Worksheet – MTH 305 (Spring 2017)

(1) Use the graph of f to state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x\to 2^-} f(x)$

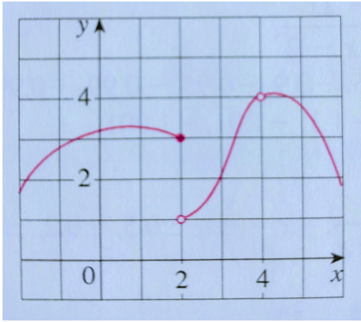
(b) $\lim_{x\to 2^+} f(x)$

(c) $\lim_{x\to 2} f(x)$

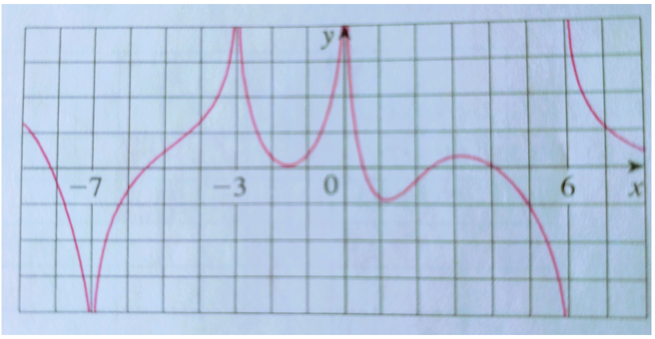
(d) $f(2)$

(e) $\lim_{x\to 4} f(x)$

(f) $f(4)$



(2) For the function f whose graph is shown, state the following.



(a) $\lim_{x\to -7} f(x)$

(b) $\lim_{x\to -3} f(x)$

(c) $\lim_{x\to 0} f(x)$

(d) $\lim_{x\to 6^-} f(x)$

(e) $\lim_{x\to 6^+} f(x)$

(3) Sketch the graph of the function and use it to determine the values of a for which $\lim_{x\to a} f(x)$ exists.

$$f(x) = \begin{cases} 1+x & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x < 1 \\ 2-x & \text{if } x \ge 1 \end{cases}$$

(4) Sketch the graph of an example of a function f that satisfies all of the given conditions.

(a) $\lim_{x\to 0^-} f(x) = -1, \quad \lim_{x\to 0^+} f(x) = 2, \quad f(0) = 1.$

(b) $\lim_{x\to 3^+} f(x) = 4, \quad \lim_{x\to 3^-} f(x) = 2, \quad \lim_{x\to -2} f(x) = 2, \quad f(3) = 3, \quad f(-2) = 1.$

(5) Let

$$g(x) = \begin{cases} x & \text{if } \; x < 1 \\ 3 & \text{if } \; x = 1 \\ 2-x^2 & \text{if } \; 1 < x \le 2 \\ x-3 & \text{if } \; x > 2 \end{cases}$$

(a) Evaluate each of the following, if it exists.

(i) $\lim_{x\to 1^-} g(x)$

(ii) $\lim_{x\to 1} g(x)$

(iii) $g(1)$

(iv) $\lim_{x\to 2^-} g(x)$

(v) $\lim_{x\to 2^+} g(x)$

(vi) $\lim_{x\to 2} g(x)$

(b) Sketch the graph of g.

(6) (a) Evaluate the function $f(x) = x^2 - (2^x/1000)$ for $x = 1, 0.8, 0.6, 0.4, 0.2, 0.1$, and 0.05, and guess the value of

$$\lim_{x\to 0}\left(x^2 - \frac{2^x}{1000}\right)$$

(b) Evaluate $f(x)$ for $x = 0.04, 0.02, 0.01, 0.005, 0.003$, and 0.001. Guess again.

(7) Evaluate the limit and justify each step by indicating the appropriate Limit Law(s).

(a) $\lim_{x\to 3}(5x^3 - 3x^2 + x - 6)$

(b) $\lim_{x\to -1}(x^4 - 3x)(x^2 + 5x + 3)$

(c) $\lim_{t\to -2}\dfrac{t^4 - 2}{2t^2 - 3t + 2}$

(d) $\lim_{u\to -2}\sqrt{u^4 + 3u + 6}$

(e) $\lim_{x\to 8}(1 + \sqrt[3]{x})(2 - 6x^2 + x^3)$

(f) $\lim_{x\to 2}\sqrt{\dfrac{2x^2 + 1}{3x - 2}}$

(8) Find $\lim_{t\to 0}\dfrac{\sqrt{t^2 + 9} - 3}{t^2}$.

(9) Use the Squeeze Theorem to show that

$$\lim_{x\to 0}(x^2\cos(20\pi x)) = 0.$$

Illustrate by graphing $f(x) = -x^2$, $g(x) = x^2\cos(20\pi x)$, and $h(x) = x^2$ on the same sheet (screen).

(10) Evaluate the limit, if it exists

(a) $\lim_{x\to 5}\dfrac{x^2 - 6x + 5}{x - 5}$

(b) $\lim_{x\to -4}\dfrac{\frac{1}{4} + \frac{1}{x}}{4 + x}$

(c) $\lim_{t\to 0}\dfrac{\sqrt{1 + t} - \sqrt{1 - t}}{t}$

(d) $\lim_{x\to -4}\dfrac{\sqrt{x^2 + 9} - 5}{x + 4}$

(e) $\lim_{h\to 0}\dfrac{(x + h)^3 - x^3}{h}$