Week 2 – Worksheet – MTH 305 (Spring 2017)

(1) For each of the following, determine whether or not the given sequence is convergent or divergent. If convergent, determine its limit. Use the properties of limits discussed in class to justify your answer.

(a) $\left\{\dfrac{2}{n^2}\right\}_{n\in\mathbb{N}}$

(b) $\left\{\dfrac{3n}{n+2}\right\}_{n\in\mathbb{N}}$

(c) $\{2-(-1)^n\}_{n\in\mathbb{N}}$

(d) $\left\{\dfrac{2-(-1)^n}{n}\right\}_{n\in\mathbb{N}}$

(2) In the following exercise, we are going to numerically approximate the area between the x-axis and the plotted curve. (see plots below)

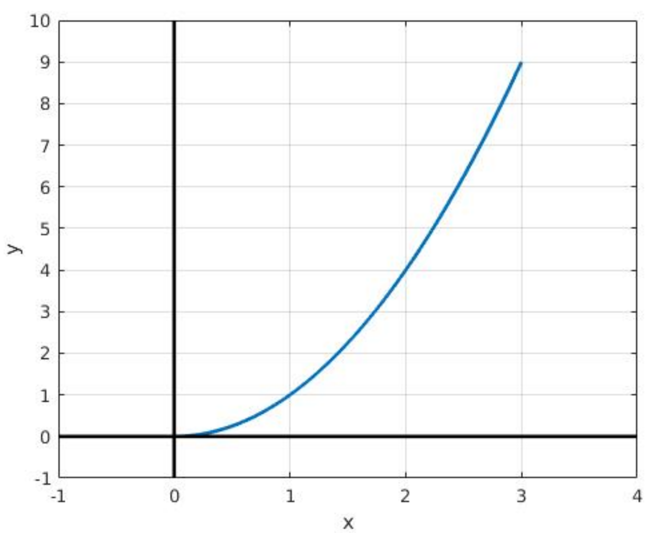
(a) Approximating area under the curve using rectangles: There are two plots under "Approximation 1". For the first plot (*approximation from below*), draw rectangles of base 1 unit and height chosen such that the top left corner of the rectangle just intersects the graph of the curve. You should notice that the sum of the area of the rectangles is *less* than the area under the given curve and all the rectangles lie below the plotted curve. In the second plot (*approximation from above*), draw and shade rectangles of base 1 unit and height chosen such that the top right corner of the rectangle intersects the graph of the curve. You should notice that the sum of the area of the rectangles is *greater* than the area under the given curve and parts of the rectangles lie above the plotted curve. Can you guess the relationship between the areas of the rectangles and the true area under the curve?

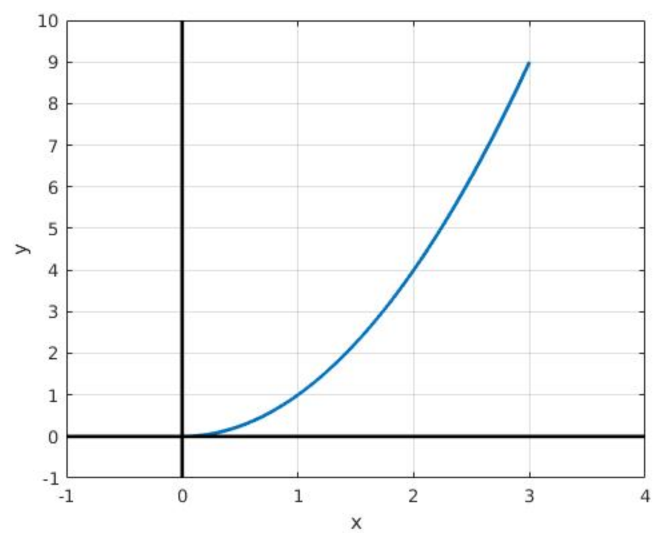
(b) Repeat the process in "Approximation 2" and "Approximation 3" with rectangles of base 0.5 and 0.25 units respectively. How does your approximation change?

(c) Can you guess the exact area under the plotted curve?

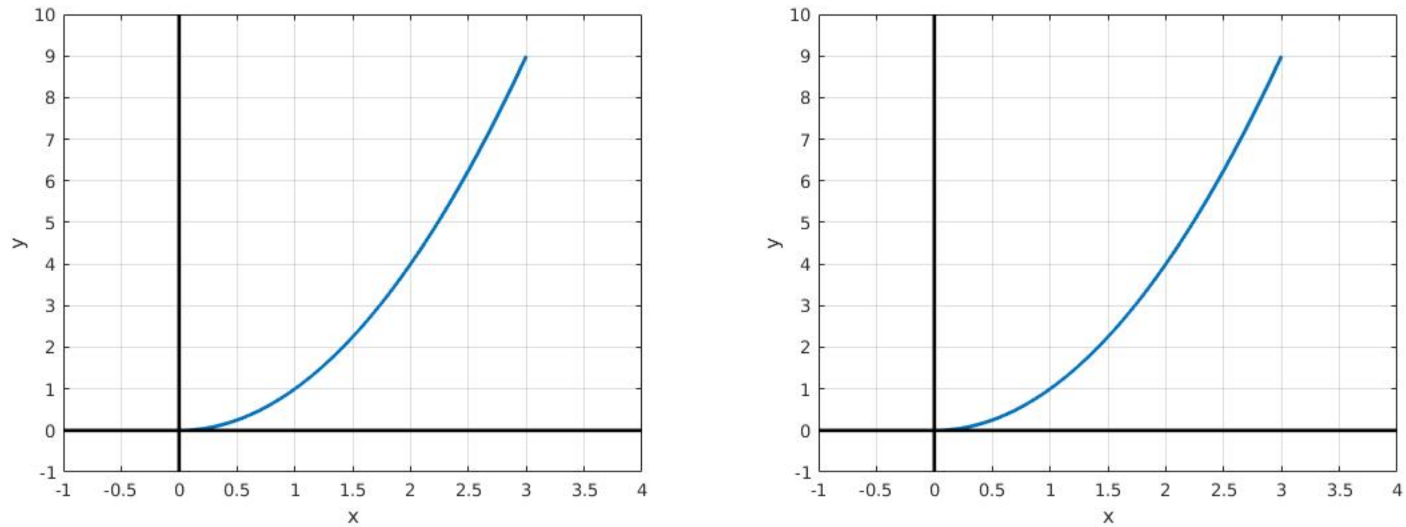
(d) Discuss how this exercise relates to sequences and properties of sequences we discussed in class.

Approximation 1

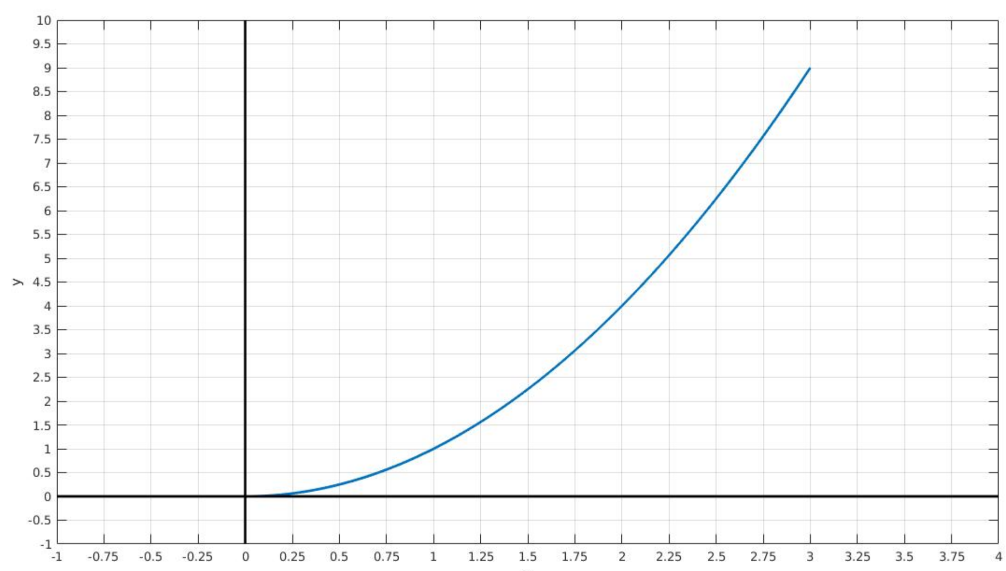




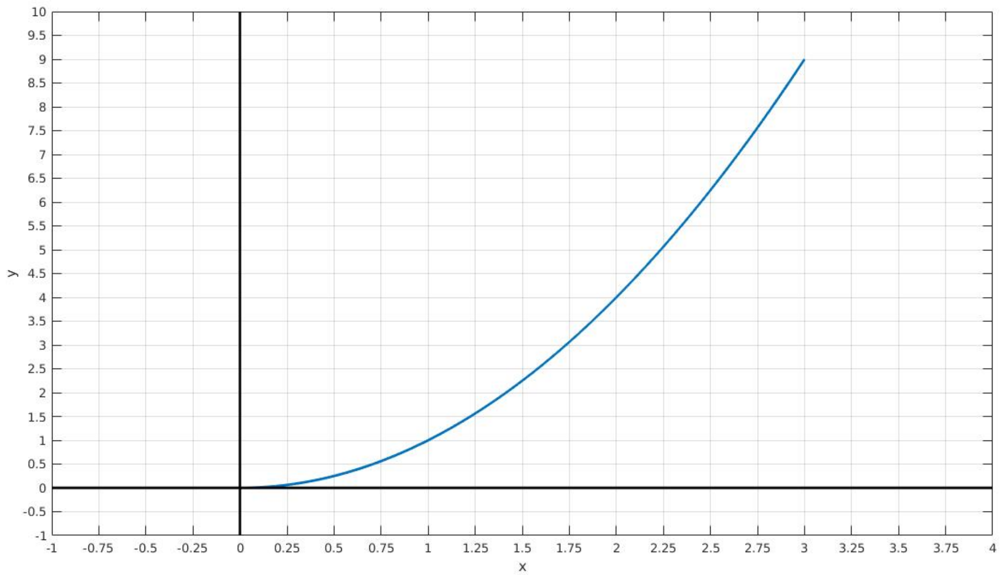
Approximation 2



Approximation 3



Approximation 3 (contd...)



(3) Decide whether the geometric series converge or diverge. Justify your answer. If the series converges, compute its sum.

(a) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$

(b) $\sum_{n=1}^{\infty} \frac{2}{3^{n-1}}$

(c) $2 - 2 + 2 - 2 + 2 - 2 \dots +$

(d) $\frac{5}{4} + \frac{25}{16} + \dots + \left(\frac{5}{4}\right)^n + \dots$

(4) (*Writing recurring decimals as fractions*) Let $x = 0.\overline{25}$. Show that $x = \frac{25}{99}$. (note: the notation $0.\overline{25}$ means that the digits 2 and 5 repeat indefinitely; i.e., $x = 0.252525252525\dots$. Hint: try and write x as a geometric series!)