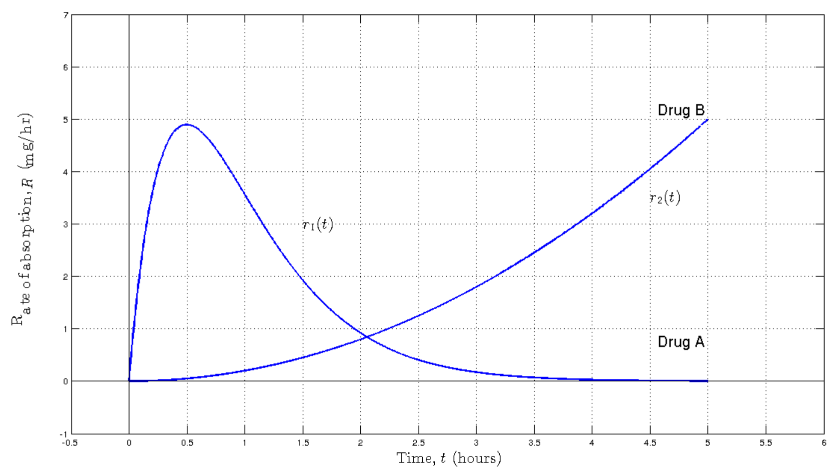
Week 16 – Worksheet – MTH 305 (Spring 2017)

(1) The rate of absorption of two drugs, in milligrams per hour, in the first 5 hours after injection is shown in the figure below.



(a) After 2 hours, which drug has been absorbed the most? At the end of 5 hours? How did you arrive at your answer?

(b) How much of Drug B has been absorbed after 4 hours? Include units. Round your answer to the nearest integer. How did you arrive at your answer?

(2) The amount of oil spilling out of a defective offshore oil rig, V, in tons per hour, is approximated by $V = 5.2e^{-0.032t}$, where t is hours since the initial breakdown of the oil rig.

(a) If it takes 4 days for the rig to be fixed, find the total volume of oil spilt. Round your answer to the nearest integer.

(b) If environmental cleanup costs are $5,600 per ton, how much does it cost the company to clean up the oil spill?

(3) Find the volume of the solid obtained by rotating about the axis the region under the curve $y = 2\sqrt{x}$ from 0 to 1. Illustrate the definition of volume by sketching a typical approximating cylinder.

(4) In the exercises below, compute the definite integral $\int_a^b f(x)dx$.

(a) $f(x) = \frac{x^2-1}{x+1}, a = 3, b = 5$

(b) $f(x) = x^3(\frac{1}{x} + 2\sqrt{x} - 1), a = 1, b = 2$

(c) $f(x) = 3(e^x - 1), a = 0, b = 1$

(5) In the exercises below, compute the given integral. Use differentiation to justify your answers.

(a) $\int \sqrt[4]{x-5}\, dx$

(b) $\int (x+3)(x^2+6x-1)^{5/7}\, dx$

(c) $\int_{-1}^{1} x\sqrt{x^2-1}(x^2-1)\, dx$

(6) Given $\int_1^2 f(x)dx = 4$, $\int_2^3 f(x)dx = 2$, $\int_1^2 g(x)dx = -1$ and $\int_2^3 g(x)dx = 1$, evaluate

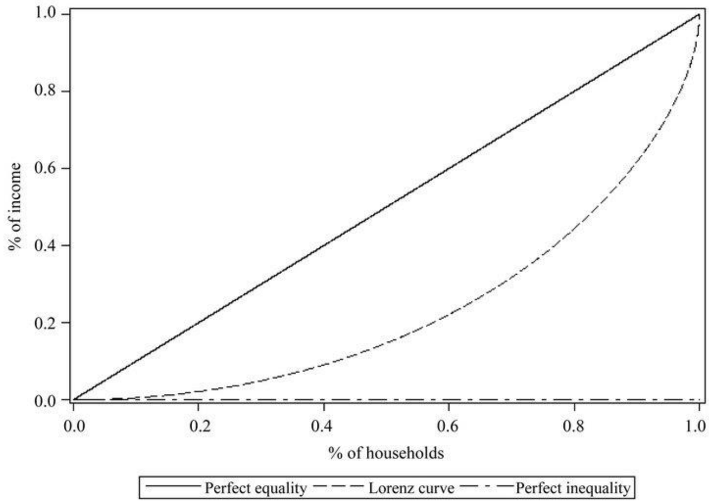
(a) $\int_1^3 f(x)dx$

(b) $\int_3^1 g(x)dx$

(c) $\int_1^3 [2f(x) - 5g(x)]dx$.

Measuring Inequality

In recent times, the national debt of Greece has spiralled alarmingly. This is a human tragedy because it harms millions of people. But will some be more affected than others? Calculus can help us investigate this situation. The inequality in the distribution of wealth in a country can be measured using the Gini Coefficient (named after Corrado Gini) which is calculated using the Lorenz Curve (named after Conrad Lorenz). The Lorenz Curve is drawn on a set of axes where the *x* axis represents the percentage of the population going from the poorest to the richest. The *y*-axis represents the percentage of the total income received.



A perfectly equitable distribution of wealth would be represented by a straight line. This would show for example that the bottom 30% of people shared in 30% of the wealth. The equation of the Lorenz curve is a polynomial. The Gini Coefficient measures the degree of income inequality and is calculated using the size of the area between the line of perfect equality and the Lorenz curve (Area A) as below:

$$\text{Gini Coefficient} = \frac{\text{Area A}}{\text{Area under line of equality}}$$

The most recent data available for Greece is shown below:

Percentage of Population	Lowest 10%	Second 10%	Third 10%	Fourth 10%	Fifth 10%	Sixth 10%	Seventh 10%	Eighth 10%	Ninth 10%	Top 10%
Percentage of wealth	3%	4%	6%	7%	8%	9%	11%	12%	15%	25%

a) Plot a graph of the Lorenz Curve for Greece.

b) Calculate an estimate for the Gini index for Greece (use for eg., trapezoidal method).

c) Find a polynomial function that models the Lorenz Curve for Greece.

d) Display the Lorenz Curve for Greece using the polynomial function model.

e) Using your polynomial function and your "by hand" skills in integration, calculate an estimate for the Gini index.

(based on exercises by Dr. Ian Thomson, Ormiston College)