# Week 15 – Worksheet – MTH 305 (Spring 2017)

(1) Find the antiderivative $F(x)$ of the function $f(x) = e^x + x - 1$ that satisfies $F(0) = 6$.

(2) Find the antiderivative $F(x)$ of the function $f(x) = 1/x^2$ that satisfies $F(-1) = 1$ and $F(2) = 0$.

(3) Keeping a constant acceleration, a motorcycle can go from 40 miles per hour to 60 miles per hour in 5 seconds.

    (a) What is the motorcycle's acceleration in miles per second squared?

    (b) What is the distance traveled by the motorcycle during these 5 seconds?

(4) In the exercises below, compute the definite integral $\int_a^b f(x)dx$.

    (a) $f(x) = 6x^5, a = -1, b = 1$

    (b) $f(x) = x^3(2x^2 - 1), a = 0, b = 1$

    (c) $f(x) = (x + 2)(3 - x^2), a = 0, b = 1$

    (d) $f(x) = 1/x^3 + 1/x^4, a = 1, b = 2$

    (e) $f(x) = \sqrt{x}(x^3 + 1), a = 0, b = 1$

    (f) $f(x) = e^{x+3}, a = -2, b = 0$

    (g) $f(x) = |x|, a = -2, b = 5$

    (h) $f(x) = |3x - 1|, a = -1, b = 0$

    (i) $f(x) = \begin{cases} x^3 & \text{if } x \le 1 \\ x^2 & \text{if } x > 1 \end{cases}$ , $a = 0, b = 2$

(5) Let $f(x) = x^3$ and consider the function $g(x) = \int_1^x f(t)dt$.

    (a) Find a formula for $g(x)$.

    (b) Evaluate $g'(x)$.

    (c) Compute $g(1)$.

    (d) Fill in the blanks:
        $g(x)$ is the _____ of $f(x) = x^3$ that takes the value _____ at $x =$ _____.

(6) Let $g_1(x) = \int_0^x \dfrac{1}{2t^2 + e^t}dt$. Then
    $g_1(x)$ is the _____ of $f_1(x) =$ _____ that takes the value _____ at $x =$ _____.

(7) In the exercises below, compute the given integral. Use differentiation to justify your answers.

    (a) $\displaystyle\int 3x^2(x^3 + 2)dx$

    (b) $\displaystyle\int x^3\sqrt{x^4 + 3}dx$

    (c) $\displaystyle\int_{-1}^1 \dfrac{x + 1/2}{(x^2 + x + 1)^5}dx$

    (d) $\displaystyle\int \dfrac{1}{x^2}e^{2/x}dx$

    (e) $\displaystyle\int_1^4 \dfrac{1}{\sqrt{x}(1 + \sqrt{x})^3}dx$

(8) $\int xe^{2x}dx$ (Hint: use integration by parts)

(9) $\int(x^2 + 2x + 3)e^x dx$ (Hint: use integration by parts)