# Week 12 – Worksheet – MTH 305 (Spring 2017)

(1) Find the derivative of $f(x) = (2x^3 + 7)(3x^2 - 2x)$ in two ways: by using the Product Rule and by performing the multiplication first. Do your answers agree?

(2) Find the derivative of the function
$$F(x) = \frac{3x^4 - 4x^3 + 3x^2 + 3\sqrt{x}}{x^3}$$
in two ways: by using the Quotient Rule and by simplifying first. Show that your answers are equivalent. Which method do you prefer?

(3) Using the rules for computing derivatives, compute the derivative of the given function. At each step, specify the formula you applied.

(a) $f(x) = 4x - 3x^2 + 7$

(b) $g(t) = \dfrac{x+6}{x+1}$

(c) $h(\mu) = (\mu + 1)(\mu + 2)(\mu + 3)$

(d) $f(p) = -\dfrac{2}{p^2} + 2^{\frac{1}{3}}$

(e) $h(t) = 3\sqrt{3t^2 + 2t + 1}$

(f) $j(x) = \sqrt[4]{t}(\sqrt[3]{t} + 2t + 1)$

(g) $g(x) = \dfrac{x^2 + 2x + 1}{3\sqrt{x}}$

(h) $h(t) = \sqrt{\dfrac{2t-3}{t+2}}$

(i) $f(x) = ax^3 + bx^2 + cx + d$

(4) Find equations of the tangent line and normal line to the curve at the given point.

(a) $y = \dfrac{2x+3}{x^2+4}$ at the point corresponding to $x = 0$.

(b) $y = x + 2\sqrt{x}$ at the point corresponding to $x = 1$.

(5) Determine the intervals where each of the following functions is increasing, and determine the intervals where it is decreasing.

(a) $f(x) = \dfrac{1}{x+3}$.

(b) $g(x) = 3x^4 - 16x^3 + 18x^2$ within the interval $[-1, 4]$.

(6) The equation of motion of a particle is $s = t^3 - 3t$, where $s$ is in meters and $t$ is in seconds. Find

(a) the velocity and acceleration as functions of $t$,

(b) the acceleration after 2s, and

(c) the acceleration when the velocity is 0.

(7) Use the chain rule to compute $(f(g(x)))'$.

(a) $f(y) = \sqrt[3]{y^4 + 6}, \quad g(x) = 10 - 4x^3 + 5x$

(b) $f(y) = \dfrac{3}{y}, \quad g(x) = x^2 - 2x$

(c) $f(y) = y^8 + 6y^3, \quad g(x) = x^2 + 3x - \sqrt{3}$

(8) Use the chain rule to compute the derivative of the given function.

(a) $f(x) = (x^2 + 2x - 3)^7$

(b) $g(x) = \sqrt[4]{x^2 + 4x^6}$

(c) $h(x) = -\sqrt{\dfrac{3x + 5}{x^2 + 2}}$

(d) $f(t) = \sin(t^2 \cos(t))$

(e) $g(t) = \left(\dfrac{x^2 - 2}{x^2 + 1}\right)^2$

(9) Find the equation of the tangent line to the graph of $f(x) = \left(\dfrac{3x}{x+4}\right)^3$ at the point $(-1, -1)$.

(10) Compute the value of $(f \circ g)'(t)$ at the given value of $t$.

$$f(y) = y^3 - 3, \quad g(t)\sqrt{t}, \quad t = 16.$$