Week 10 – Worksheet – MTH 305 (Spring 2017)

(1) Find the equation of the tangent line to the curve at the given point.

(a) $y = 4x - 3x^2$ at $(2, -4)$

(b) $y = x^3 - 3x + 1$ at $(2, 3)$

(c) $y = \sqrt{x}$ at $(1, 1)$

(d) $y = \dfrac{2x+1}{x+2}$ at $(1, 1)$

(2) Each limit represents the derivative of some function f at some number a. State such an f and a in each case.

(a) $\lim_{h \to 0} \dfrac{(1+h)^{10} - 1}{h}$

(b) $\lim_{h \to 0} \dfrac{\sqrt[4]{16+h} - 2}{h}$

(c) $\lim_{x \to 5} \dfrac{2^x - 32}{x - 5}$

(d) $\lim_{x \to \pi/4} \dfrac{\tan x - 1}{x - \pi/4}$

(e) $\lim_{h \to 0} \dfrac{\cos(\pi + h) + 1}{h}$

(3) The cost (in dollars) of producing x units of a certain commodity is $C(x) = 5000 + 10x + 0.05x^2$.

(a) Find the average rate of change of C with respect to x when the production level is changed

(i) from $x = 100$ to $x = 105$

(ii) from $x = 100$ to $x = 101$

(b) Find the instantaneous rate of change of C with respect to x when $x = 100$ (This is called the *marginal cost*).

(4) The number N of locations of a popular coffeehouse chain is given by the table.

Year	2004	2005	2006	2007	2008
N	8569	10,241	12,440	15,011	16,680

(a) Find the average rate of growth

(i) from 2006 to 2008

(ii) from 2006 to 2007

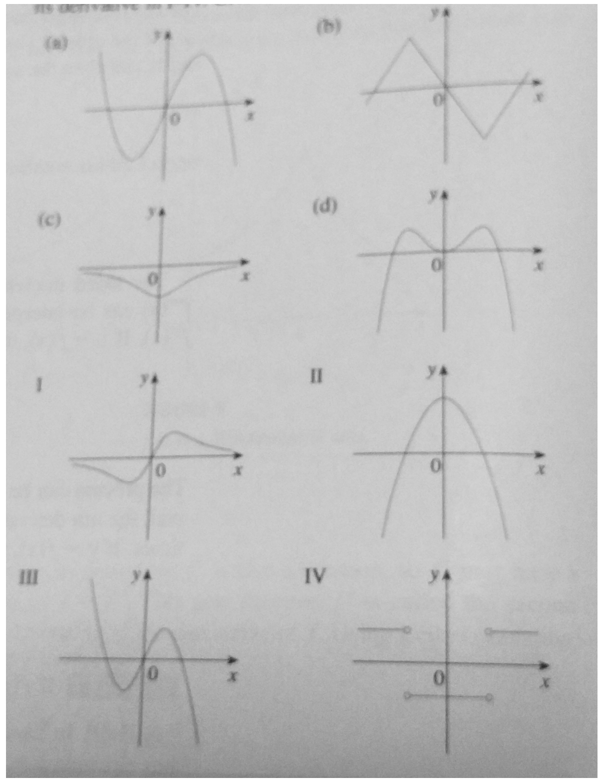
(iii) from 2005 to 2006

In each case, include the units.

(b) Estimate the instantaneous rate of growth in 2006 by taking the average of two average rates of change. What are its units?

(c) Estimate the instantaneous rate of growth in 2007 and compare it with the growth rate in 2006. What do you conclude?

(5) Match the graph of each function in (a)–(d) with the graph of the derivative in I–IV. Give reasons for your choices.



(6) The displacement (in meters) of a particle moving in a straight line is given by $s = t^2 - 8t + 18$, where t is measured in seconds.

(a) Find the average velocity over each time interval:
(i) $[3, 4]$ (ii) $[3.5, 4]$ (iii) $[4, 5]$ (ii) $[4, 4.5]$

(b) Find the instantaneous velocity when $t = 4$.

(c) Draw the graph of s as a function of t and draw the secant lines whose slopes are the average velocities in part (a) and the tangent line whose slope is the instantaneous velocity in part (b).