Week 1 – Worksheet – MTH 305 (Spring 2017)

(1) Determine direct and recursive formulas for the sequences whose first few terms coincide with the given ones:

(a) $5, 14, 23, 32, 41, \ldots$

(b) $3, \frac{7}{2}, 4, \frac{9}{2}, 5, \ldots$

(c) $-\frac{21}{4}, -\frac{11}{2}, -\frac{23}{4}, -6, -\frac{25}{4}, \ldots$

(d) $\frac{e}{2}, 0, -\frac{e}{2}, -e, -\frac{3e}{2}, \ldots$

(e) $3, 12, 48, 192, 768, \ldots$

(f) $-13, 26, -52, 104, \ldots$

(g) $1, 2, 6, 15, 31, \ldots$ (only provide a recursive formula)

(h) $0, 1, 2, 5, 12, 29, \ldots$ (only provide a recursive formula)

(2) For each of the following, determine whether or not they converge. If they converge, what is the limit? Provide some algebraic justification.

(a) $\left\{\frac{3n+1}{7n-4}\right\}_{n\in\mathbb{N}}$

(b) $\left\{\sin\left(\frac{n\pi}{4}\right)\right\}_{n\in\mathbb{N}}$

(c) $\left\{\left(1+\frac{1}{n}\right)^2\right\}_{n\in\mathbb{N}}$

(d) $\{(-1)^n n\}_{n\in\mathbb{N}}$

(e) $\left\{\sqrt{n^2+1}-n\right\}_{n\in\mathbb{N}}$

(f) $\left\{\frac{n}{n+1}\right\}_{n\in\mathbb{N}}$

(g) $\left\{3+\frac{(-1)^n 2}{n}\right\}_{n\in\mathbb{N}}$

(h) $\left\{\frac{n^2-2n+1}{n-1}\right\}_{n\in\mathbb{N}}$

(3) Determine if the following sequences are bounded. Briefly justify your answers.

(a) $\{2n\}_{n\in\mathbb{N}}$

(b) $\{1+(-1)^n(2n-1)\}_{n\in\mathbb{N}}$

(c) $\left\{\frac{n}{n+3}\right\}_{n\in\mathbb{N}}$

(4) Let S be a positive real number and consider the recursive definition of the sequence,

Choose x_0 to be a positive real number, and let

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{S}{x_n}\right).$$

(a) Make and fill out a table with columns n and x_n for $n = 1, 2, \ldots, 5$. (try $S = 4$ and $S = 9$, although any positive number S will work)

(b) Can you guess the value of $L = \lim_{n\to\infty} x_n$?

(c) Repeat your experiment with a different first term, x_0. What effect does this have on the sequence?

(d) Can you comment on how "fast" the sequence approaches L?

(5) Write out a careful proof to justify that

$$\lim_{n\to\infty} \frac{2n-3}{3n+7} = \frac{2}{3}.$$