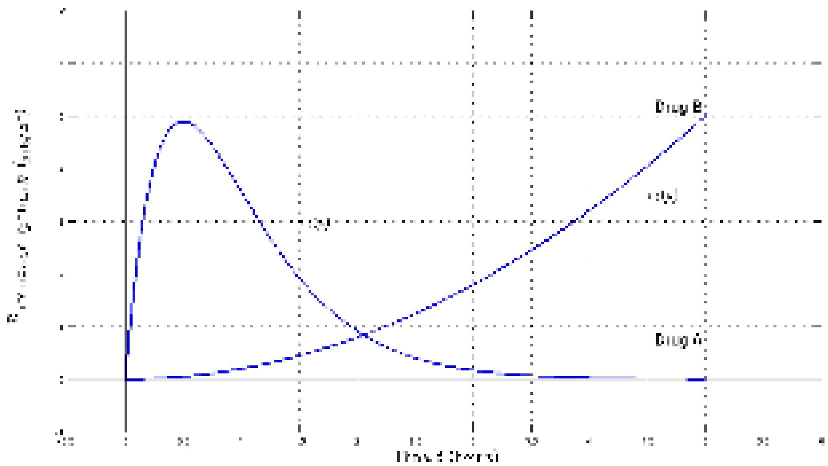
Week 16 – Worksheet – MTH 305 (Spring 2017)

(1) The rate of absorption of two drugs, in milligrams per hour, in the first 5 hours after injection is shown in the figure below.



Drug A Drug B

- (a) After 2 hours, which drug has been absorbed the most? At the end of 5 hours? How did you arrive at your answer? area under curve/count grid squares.
- (b) How much of Drug B has been absorbed after 4 hours? Include units. Round your answer to the nearest integer. How did you arrive at your answer? area under $r_2(t) \approx 8.25$ grid squares each grid square = 0.5mg
  4 mg of Drug B absorbed

(2) The amount of oil spilling out of a defective offshore oil rig, V, in tons per hour, is approximated by $V = 5.2e^{-0.032t}$, where t is hours since the initial breakdown of the oil rig.

- (a) If it takes 4 days for the rig to be fixed, find the total volume of oil spilt. Round your answer to the nearest integer. $\int_0^{96} v(t)dt \approx 155$ tons
- (b) If environmental cleanup costs are \$5,600 per ton, how much does it cost the company to clean up the oil spill? 155 × \$5600 = \$868,000

(3) Find the volume of the solid obtained by rotating about the axis the region under the curve $y = 2\sqrt{x}$ from 0 to 1. Illustrate the definition of volume by sketching a typical approximating cylinder.

(4) In the exercises below, compute the definite integral $\int_a^b f(x)dx$.

- (a) $f(x) = \frac{x^2-1}{x+1}, a = 3, b = 5$
- (b) $f(x) = x^3(\frac{1}{x} + 2\sqrt{x} - 1), a = 1, b = 2$
- (c) $f(x) = 3(e^x - 1), a = 0, b = 1$

(see pp. 2)

(5) In the exercises below, compute the given integral. Use differentiation to justify your answers.

- (a) $\int \sqrt[4]{x-5}\, dx$
- (b) $\int (x+3)(x^2+6x-1)^{5/7}\, dx$
- (c) $\int_{-1}^{1} x\sqrt{x^2-1}(x^2-1)\, dx$

(see pp. 3)

(6) Given $\int_1^2 f(x)dx = 4$, $\int_2^3 f(x)dx = 2$, $\int_1^2 g(x)dx = -1$ and $\int_2^3 g(x)dx = 1$, evaluate

- (a) $\int_1^3 f(x)dx$ = 6
- (b) $\int_3^1 g(x)dx$ = 0
- (c) $\int_1^3 [2f(x) - 5g(x)]dx.$ = 12

(Similar to HW 6 #1)

#4 (a) $\int_3^5 \frac{x^2-1}{x+1}\,dx = \int_3^5 \frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}}\,dx = \int_3^5 (x-1)\,dx$

$= \frac{x^2}{2}\Big|_3^5 - x\Big|_3^5 = \frac{1}{2}(25-9) - (5-3)$

$= 8-2 = \boxed{6}$

(b) $\int_1^2 x^3\left(\frac{1}{x}+2\sqrt{x}-1\right)dx = \int_1^2 \left(x^2+2x^{7/2}-x^3\right)dx$

$= \int_1^2 x^2\,dx + 2\int_1^2 x^{7/2}\,dx - \int_1^2 x^3\,dx$

$= \frac{x^3}{3}\Big|_1^2 + 2\left(\frac{x^{9/2}}{9/2}\Big|_1^2\right) - \left(\frac{x^4}{4}\Big|_1^2\right)$

$= \frac{1}{3}(8-1) + \frac{4}{9}\left(2^{9/2}-1\right) - \frac{1}{4}(16-1)$

$= \frac{7}{3} + \frac{64\sqrt{2}}{9} - \frac{4}{9} - \frac{15}{4} = \frac{84+256\sqrt{2}-16-135}{36}$

$= \boxed{\frac{256\sqrt{2}-67}{36}}$

(c) $\int_0^1 3(e^x-1)\,dx = 3\int_0^1 e^x\,dx - 3\int_0^1 dx = 3\left(e^x\Big|_0^1\right) - 3\left(x\Big|_0^1\right)$

$= 3(e-1) - 3(1-0)$

$= \boxed{3e-6}$

#5 (a) $\int \sqrt[4]{x-5}\,dx$

Let $u(x) = x-5$. Then $\frac{du}{dx} = 1$, or, $du = dx$

$$\int \sqrt[4]{x-5}\,dx = \int \sqrt[4]{u}\,du = \int u^{1/4}du = \frac{u^{5/4}}{5/4} + C = \boxed{\frac{4}{5}(x-5)^{5/4} + C}$$

(b) $\int (x+3)(x^2+6x-1)^{5/7}dx$

Let $u(x) = x^2+6x-1$. Then $\frac{du}{dx} = 2x+6 = 2(x+3)$. Then $(x+3)\,dx = \frac{1}{2}du$.

$$\int (x+3)(x^2+6x-1)^{5/7}dx = \frac{1}{2}\int u^{5/7}du = \frac{1}{2}\,\frac{u^{12/7}}{12/7} + C = \boxed{\frac{7}{24}(x^2+6x-1)^{12/7} + C.}$$

(c) $\int_{-1}^{1} x\sqrt{x^2-1}\,(x^2-1)\,dx$

Let $u(x) = x^2-1$. Then $\frac{du}{dx} = 2x$, or, $x\,dx = \frac{1}{2}du$.

when $x=-1$, $u=0$

$x=1$, $u=0$

Hence $\int_{-1}^{1} x\sqrt{x^2-1}\,(x^2-1)\,dx = \frac{1}{2}\int_0^0 \sqrt{u}\,u\,du = \boxed{0.}$ (prop. #23(a))