Week 15 – Worksheet – MTH 305 (Spring 2017)

(1) Find the antiderivative $F(x)$ of the function $f(x) = e^x + x - 1$ that satisfies $F(0) = 6$.

(2) Find the antiderivative $F(x)$ of the function $f(x) = 1/x^2$ that satisfies $F(-1) = 1$ and $F(2) = 0$.

see class notes

(3) Keeping a constant acceleration, a motorcycle can go from 40 miles per hour to 60 miles per hour in 5 seconds.

(a) What is the motorcycle's acceleration in miles per second squared? $1/900$ m/s^2

(b) What is the distance traveled by the motorcycle during these 5 seconds? $\frac{5}{72}$ miles

(4) In the exercises below, compute the definite integral $\int_a^b f(x)dx$.

(a) $f(x) = 6x^5, a = -1, b = 1$ 0

(b) $f(x) = x^3(2x^2 - 1), a = 0, b = 1$ see class notes

(c) $f(x) = (x + 2)(3 - x^2), a = 0, b = 1$ $79/12$

(d) $f(x) = 1/x^3 + 1/x^4, a = 1, b = 2$ $2/3$

(e) $f(x) = \sqrt{x}(x^3 + 1), a = 0, b = 1$ $8/9$

(f) $f(x) = e^{x+3}, a = -2, b = 0$ $e(e^2-1)$

(g) $f(x) = |x|, a = -2, b = 5$ see class notes

(h) $f(x) = |3x - 1|, a = -1, b = 0$ $5/2$

(i) $f(x) = \begin{cases} x^3 & \text{if} \quad x \leq 1 \\ x^2 & \text{if} \quad x > 1 \end{cases}, a = 0, b = 2$ $\frac{31}{12}$

(5) Let $f(x) = x^3$ and consider the function $g(x) = \int_1^x f(t)dt$.

(a) Find a formula for $g(x)$. $g(x) = \int_1^x t^3\,dt = \frac{t^4}{4}\Big|_1^x = \frac{x^4}{4} - \frac{1}{4}$

(b) Evaluate $g'(x)$. $g'(x) = \frac{d}{dx}\left(\frac{x^4}{4} - \frac{1}{4}\right) = x^3$

(c) Compute $g(1)$. $g(1) = \frac{1^4}{4} - \frac{1}{4} = 0$

(d) Fill in the blanks:
$g(x)$ is the antiderivative of $f(x) = x^3$ that takes the value 0 at $x =$ 1.

(6) Let $g_1(x) = \displaystyle\int_0^x \frac{1}{2t^2 + e^t} dt$. Then
$g_1(x)$ is the antiderivative of $f_1(x) =$ $\frac{1}{2x^2+e^x}$ that takes the value 0 at $x =$ 0.

(7) In the exercises below, compute the given integral. Use differentiation to justify your answers.

(a) $\displaystyle\int 3x^2(x^3 + 2)dx$ $\frac{x^6}{2} + 2x^3 + C$ (Substitution: $u(x) = x^3+2$)

(b) $\displaystyle\int x^3\sqrt{x^4 + 3}dx$

(c) $\displaystyle\int_{-1}^{1} \frac{x + 1/2}{(x^2 + x + 1)^5} dx$

see class notes

(d) $\displaystyle\int \frac{1}{x^2} e^{2/x} dx$ $\frac{-e^{2/x}}{2} + C$ (Substitution: $u(x) = \frac{2}{x}$)

(e) $\displaystyle\int_1^4 \frac{1}{\sqrt{x}(1 + \sqrt{x})^3} dx$ $\frac{5}{36}$ (Substitution $u(x) = 1+\sqrt{x}$)

(8) $\int xe^{2x}dx$ (Hint: use integration by parts) $\frac{1}{4}e^{2x}(2x-1) + C$

(9) $\int(x^2 + 2x + 3)e^x dx$ (Hint: use integration by parts)

(see next page)

#9 $\displaystyle\int \underbrace{(x^2+2x+3)}_{u}\underbrace{e^x}_{v'}\,dx = (x^2+2x+3)e^x - \int \underbrace{(2x+2)}_{u}\underbrace{e^x}_{v'}\,dx$ (integration by parts)

$$= (x^2+2x+3)e^x - \left[(2x+2)e^x - \int 2e^x\,dx\right]$$

(integration by parts for 2nd term)

$$= (x^2+2x+3)e^x - (2x+2)e^x + 2\int e^x\,dx$$

$$= (x^2+2x+3)e^x - (2x+2)e^x + 2e^x + C$$

$$= e^x\left(x^2+\cancel{2x}+3-\cancel{2x}-\cancel{2}+\cancel{2}\right) + C$$

$$= (x^2+3)e^x + C$$