SELECTED SOLUTIONS – WEEK 5

#1 Determine if the given equation describes y as a function of x.

(a) $x = y^2 - 2$ $\boxed{\text{No}}$

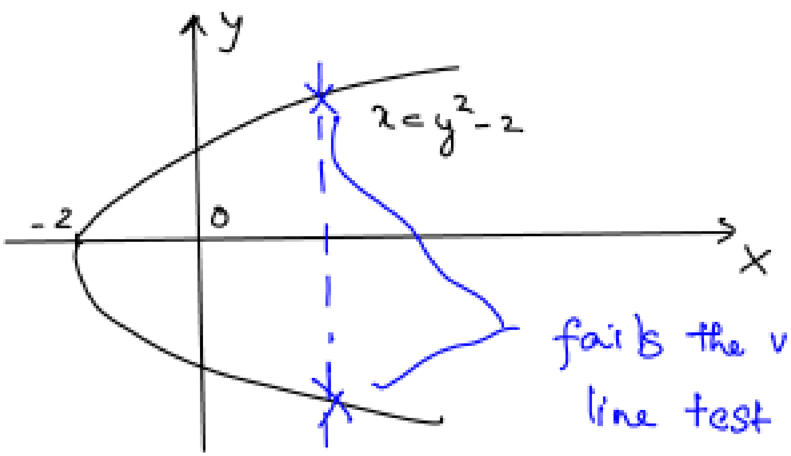
We have

$$y^2 = x + 2$$

$$\Rightarrow \quad y = \pm\sqrt{x+2}$$

For example, both $y = +2$ and $y = -2$ satisfy the given equation for $x = 2$.
There are multiple (output) values of y for the same (input) value of x.
This violates the definition of a function

Also, here is a plot of y vs. x.



fails the vertical line test

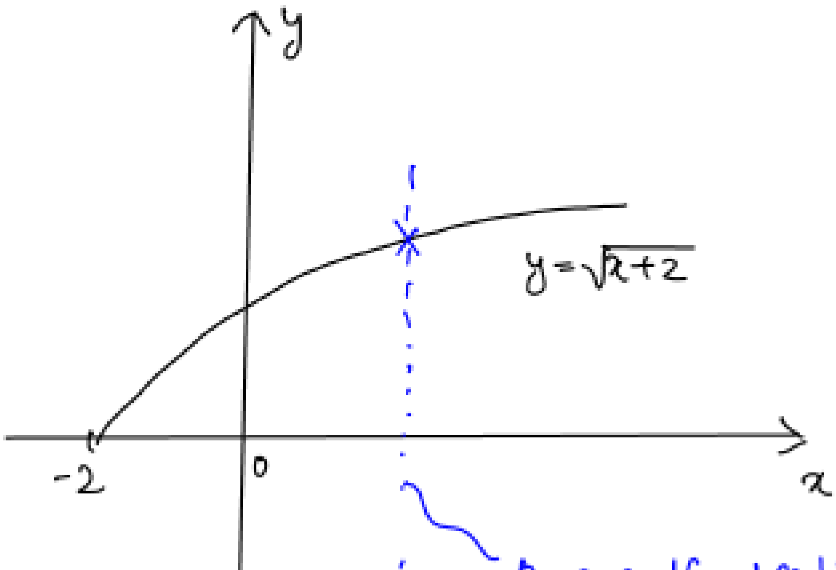
(Recall)

<u>Defn</u>: Suppose X and Y are sets.
A function from X to Y is an association or rule between the members of the sets. More precisely, for every element of X, there is a <u>unique</u> element of Y.

(b) $y = \sqrt{x+2}$ $\boxed{\text{Yes}}$

Every $x \in \mathbb{R}$ with $x \geq -2$ is assigned to a <u>unique</u> $y \in \mathbb{R}$.

Here is a plot of y vs. x



passes the vertical line test

(c) $y^2 = x^2 + 2$ $\boxed{\text{No}}$

(d) $y = 7$ $\boxed{\text{Yes}}$

(e) $4y^5 - 5x = 20$ $\boxed{\text{Yes}}$

(f) $y = \begin{cases} x+2 & \text{if } x < 0 \\ 1-x & \text{if } x \geq 0 \end{cases}$ $\boxed{\text{Yes}}$

(Graphs)

(7) $\boxed{\text{No}}$

(8) $\boxed{\text{Yes}}$

(9) $\boxed{\text{Yes}}$

(10) $\boxed{\text{No}}$

#2 Determine the domain of the given function

(a) $f(x) = \dfrac{x+4}{x^2-9}$

factoring the denominator, $f(x) = \dfrac{x+4}{(x+3)(x-3)}$

$f(x)$ is not defined when $x = -3$ and $x = 3$.

Hence, domain of f is $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$.

(equivalently $\mathbb{R} \setminus \{-3, 3\}$)

(c) $h(x) = \dfrac{1}{\sqrt[4]{x^2 - 5x}}$

rewriting the denominator, we have $h(x) = \dfrac{1}{\sqrt[4]{x(x-5)}}$

* $h(x)$ is not defined when $x=0$ and $x=5$.
* In addition, for $0 < x < 5$, $x(x-5) < 0$, and, hence $\sqrt[4]{x(x-5)}$ is not a real number

Therefore, domain of h is $(-\infty, 0) \cup (5, \infty)$.

#3 Determine the assignment rule and the domain of the specified composite functions.

(e) $f\circ g$, $g\circ f$, $f\circ f$ and $g\circ g$ if $f(x) = 1 - \frac{1}{x}$ and $g(x) = \frac{x+1}{x+2}$.

$\boxed{f\circ g}$ Assignment rule $(f\circ g)(x) = f(g(x)) = f\left(\frac{x+1}{x+2}\right) = 1 - \frac{1}{\left(\frac{x+1}{x+2}\right)} = 1 - \frac{x+2}{x+1}$

or, $\boxed{(f\circ g)(x) = \frac{-1}{x+1}}$

Domain

* we require x to be in the domain of g

  hence, we require $x \neq -2$.

* we also require $g(x)$ to be in the domain of f

  since domain$(f) = \mathbb{R}\setminus\{0\}$, we require $g(x) \neq 0$

  $\Rightarrow \frac{x+1}{x+2} \neq 0$

  $\Rightarrow x \neq -1$.

* Therefore, domain of $f\circ g$ is $\boxed{\mathbb{R}\setminus\{-2,-1\}}$

#3 Determine the assignment rule and the domain of the specified composite functions.

(e) $f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$ if $f(x) = 1 - \frac{1}{x}$ and $g(x) = \frac{x+1}{x+2}$.

$\boxed{g \circ f}$ $(g \circ f)(x) = g(f(x)) = g\left(1 - \frac{1}{x}\right) = \frac{\left(1-\frac{1}{x}\right) + 1}{\left(1-\frac{1}{x}\right) + 2} = \frac{2 - 1/x}{3 - 1/x} = \frac{2x-1}{3x-1}$

domain $= \mathbb{R} \setminus \{0, 1/3\}$

$\boxed{f \circ f}$ $(f \circ f)(x) = \frac{-1}{x-1}$ domain $= \mathbb{R} \setminus \{0, 1\}$

$\boxed{g \circ g}$ $(g \circ g)(x) = \frac{2x+3}{3x+5}$ domain $= \mathbb{R} \setminus \{-2, -5/3\}$