Question 1. Find the equation of the tangent line to the curve $y = \dfrac{3x-2}{x^2+1}$ at the point corresponding to $x = -1$. *(3 points)*

We note that when $x=-1$, $y = \dfrac{3(-1)-2}{(-1)^2+1} = \dfrac{-3-2}{1+1} = -\dfrac{5}{2}$.

Hence, we want to find the tangent line to the curve

$f(x) = \dfrac{3x-2}{x^2+1}$ at the point $\boxed{\left(-1, -\frac{5}{2}\right).}$

We next calculate the slope of the tangent line.

We have slope, $m = f'(x)\Big|_{\text{at } x=-1}$

Using the quotient rule, $f'(x) = \dfrac{(x^2+1)\frac{d}{dx}(3x-2) - (3x-2)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$

$$= \frac{(x^2+1)(3) - (3x-2)(2x)}{(x^2+1)^2}$$

$$= \frac{3x^2+3-6x^2+4x}{(x^2+1)^2}$$

$$f'(x) = \frac{-3x^2+4x+3}{(x^2+1)^2}$$

Slope at $x=-1$ $= f'(-1) = \dfrac{-3(-1)^2+4(-1)+3}{((-1)^2+1)^2} = \dfrac{-4}{4}$, or $\boxed{m=-1.}$

Using the point-slope form of a line, we have equation of tangent line at $x=-1$ is

$y - \left(\frac{-5}{2}\right) = (-1)\left(x-(-1)\right)$, or $\boxed{x+y+\frac{7}{2}=0.}$

Question 2. Using the rules for computing derivatives, compute the derivative of the given function. At each step, specify the formula/rule you applied. *(2 points each)*

(a) $g(t) = \cos(t^2\cos(t) + \sin(t))$

We have $g(t) = (f_1 \circ f_2)(t)$ where $f_1(u) = \cos(u)$ and $f_2(t) = t^2\cos(t) + \sin(t)$.

note that $f_1'(u) = -\sin(u)$ and $f_2'(t) = \frac{d}{dt}\left[t^2\cos(t)\right] + \frac{d}{dt}\left[\sin(t)\right]$ (sum rule)

$$= t^2\frac{d}{dt}\left[\cos(t)\right] + \cos(t)\frac{d}{dt}\left[t^2\right] + \cos(t) \quad \text{(product rule)}$$

$$= -t^2\sin(t) + 2t\cos(t) + \cos(t)$$

$$f_2'(t) = -t^2\sin(t) + (2t+1)\cos(t).$$

Now applying chain rule,

$$g'(t) = f_1'(f_2(t)) \cdot f_2'(t) = -\sin\left[t^2\cos(t) + \sin(t)\right] \cdot \left[-t^2\sin(t) + (2t+1)\cos(t)\right].$$

(b) $f(x) = \sqrt[3]{(x+1)(3x+2)}$

We have $f(x) = (g_1 \circ g_2)(x)$ where $g_1(u) = \sqrt[3]{u}$ and $g_2(x) = (x+1)(3x+2)$

Moreover, $g_1'(u) = \frac{1}{3}u^{-2/3}$ (by applying power rule), and

$$g_2'(x) = (x+1)\frac{d}{dx}(3x+2) + (3x+2)\frac{d}{dx}(x+1) \quad \text{(product rule)}$$

$$= (x+1)(3) + (3x+2)\cdot 1$$

$$g_2'(x) = 6x+5.$$

Now applying chain rule,

$$f'(x) = g_1'(g_2(x)) \cdot g_2'(x) = \frac{1}{3}\left[(x+1)(3x+3)\right]^{-2/3} \cdot (6x+5).$$

Question 3. The equation of motion of a particle moving is given by $s = 2t^2 - 12t + 7$, where s is in meters and t is in seconds. *(1 point each)*

(a) Find the average velocity over the following time intervals: $[4, 5]$ and $[5, 6]$.

Average velocity in time interval $[4,5]$ $= \dfrac{s(5) - s(4)}{5-4} = \dfrac{[2(5^2) - 12(5) + 7] - [2(4^2) - 12(4) + 7]}{1}$

$= -3 - (-9) = \boxed{6 \text{ m/s}}$

Average velocity in time interval $[5,6]$ $= \dfrac{s(6) - s(5)}{6-5} = \dfrac{[2(6^2) - 12(6) + 7] - (-3)}{1}$

$= 7 + 3 = \boxed{10 \text{ m/s.}}$

(b) Find the instantaneous velocity when $t = 5$ seconds.

Since the instantaneous velocity is the same as the derivative, we have

$v(t) = s'(t) = 4t - 12.$

Inst. velocity at $t = 5s$ $= v(5) = 20 - 12 = \boxed{8 \text{ m/s.}}$

(c) Find the acceleration when the velocity is 0 m/s.

We have $v(t) = 4t - 12$.

Setting $v(t) = 0$, we get $4t - 12 = 0$, or $\underline{t = 3s}$.

Now, acceleration, $a(t) = v'(t) = \boxed{4 \text{ m/s}^2}$

<u>Note</u>: acceleration is independent of time

acceleration at $t = 3s$ is 4 m/s^2.
(when velocity is 0 m/s)