#7 (b)

$$\int x^3 \sqrt{x^4+3}\, dx$$

METHOD OF SUBSTITUTION

Let $u(x) = x^4 + 3$

Now, $u'(x) = \frac{du}{dx} = 4x^3$

Hence, $4x^3\, dx = du$, or $x^3\, dx = \frac{1}{4}\, du$

Substitute in the integral

$$\int x^3 \sqrt{x^4+3}\, dx = \int \sqrt{u}\, \frac{1}{4}\, du = \frac{1}{4} \int \sqrt{u}\, du = \frac{1}{4}\left(\frac{u^{3/2}}{3/2}\right) + C$$

$$= \frac{1}{6} (x^4+3)^{3/2} + C$$

(c) $\displaystyle\int_{-1}^{1} \frac{x+1/2}{(x^2+x+1)^5}\, dx$

Let $u(x) = x^2+x+1$

then $u'(x) = \frac{du}{dx} = 2x+1 = 2(x+1/2)$

$\Rightarrow (x+1/2)\, dx = \frac{1}{2} du$

when $x=-1$, $u(-1) = (-1)^2 + (-1) + 1 = 1$

$x=1$, $u(1) = 1^2 + 1 + 1 = 3$

the original integral becomes $\displaystyle\int_{-1}^{1} \frac{x+1/2}{(x^2+x+1)^5}\, dx = \int_{1}^{3} \frac{1/2\, du}{u^5}$

$= \frac{1}{2}\displaystyle\int_{1}^{3} u^{-5}\, du$

$= \frac{1}{2}\left[\frac{u^{-4}}{-4} \Big|_1^3 \right]$ (power rule)

$= \frac{10}{81}$

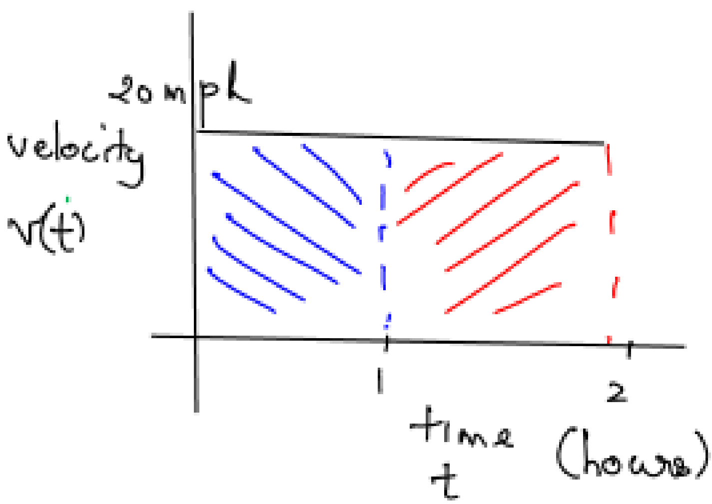
Note:

If $g(x) \le f(x)$ for $a \le x \le b$, then

$$\text{Area between } f(x) \text{ and } g(x) = \int_a^b \left[f(x) - g(x)\right] dx$$

Area under the graph

Consider



What is the distance covered by a car traveling with velocity as shown alongside?

(a) in 1 hr = 20 mph × 1 hr = 20 miles (blue shaded area in fig)

(b) in 2 hr = 20 mph × 2 hrs = 40 miles (blue + red shaded area)

In general area under curve/graph $f(x)$ in $[a,b]$

$$= \int_a^b f(x)\,dx$$