LECTURE 26 4/18/2017

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#2 $f(x) = \frac{1}{x^2}$ (given)

Want: antiderivative $F(x)$ with $F(-1)=1$ and $F(2)=0$.

Note: domain of f is $\mathbb{R}\setminus\{0\}$ or $(-\infty,0)\cup(0,\infty)$

$$F(x) = \begin{cases} -\frac{1}{x} + C_1, & \text{if } x>0 \\ -\frac{1}{x} + C_2, & \text{if } x<0 \end{cases}$$ (by the power rule)

Using $F(-1)=1$ and $F(2)=0$, we can find that

$$F(x) = \begin{cases} -\frac{1}{x} + \frac{1}{2}, & x>0 \\ -\frac{1}{x}, & x<0 \end{cases}$$

DEFINITE INTEGRALS

- We are interested in variation of a function's antiderivative over a bounded interval.

If f is a function defined on a bounded interval $[a,b]$, then the variation or change of f over $[a,b]$ is $f(b)-f(a)$, often denoted as follows

$$f(x)\Big|_a^b = f(b) - f(a)$$

$\boxed{\text{Def}^n}$ Assume that $F(x)$ is an antiderivative of $f(x)$ on the interval $[a,b]$.

The number $F(b)-F(a)$ (which depends on f, a, and b, but not on the particular antiderivative F chosen to evaluate it) is called the definite integral of f from a to b and is denoted by $\int_a^b f(x)\,dx$.

Hence, $$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a).$$

upper limit of integration

lower limit of integration

Properties

Properties

Here a, b, c are fixed real numbers, k is a constant and f, g are integrable functions defined on the interval containing a, b, c.

① $\int_a^a f(x)\,dx = 0.$

② $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

③ $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

④ $\int_a^b k f(x)\,dx = k\int_a^b f(x)\,dx$

⑤ $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

Examples:

#4 (b) $\int^1 x^3(2x^2-1)\,dx = \int^1 (2x^5 - x^3)\,dx$

#4 (b) $\displaystyle\int_0^1 x^3(2x^2-1)\,dx = \int_0^1 (2x^5 - x^3)\,dx$

$\displaystyle = 2\int_0^1 x^5\,dx - \int_0^1 x^3\,dx$ (using const. multiple, sum/difference rules)

$\displaystyle = 2\left[\frac{x^6}{6}\Big|_0^1\right] - \left[\frac{x^4}{4}\Big|_0^1\right]$ (power rule)

$\displaystyle = \frac{2}{6}(1^6 - 0^6) - \frac{1}{4}(1^4 - 0^4)$

$\displaystyle = \frac{1}{3} - \frac{1}{4}$

$\displaystyle \int_0^1 x^3(2x^2-1)\,dx = \frac{1}{12}.$

(g) $\displaystyle\int_{-2}^{5} |x|\,dx = \int_{-2}^{0} |x|\,dx + \int_0^5 |x|\,dx$ (property #3)

$\displaystyle = \int_{-2}^{0} (-x)\,dx + \int_0^5 x\,dx$ (using def^n of $|x|$)

$\displaystyle = -\int_{-2}^{0} x\,dx + \int_0^5 x\,dx$

note:

$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0. \end{cases}$

$= \left[\ldots \Big|^0 \right] + \left[x2 \Big|^5 \right]$

$$= -\left[\frac{x^2}{2}\Big|_{-2}^{0}\right] + \left[\frac{x^2}{2}\Big|_{0}^{5}\right]$$

$$= -\frac{1}{2}\left(0 - (-2)^2\right) + \frac{1}{2}\left(5^2 - 0\right)$$

$$= -\frac{1}{2}(-4) + \frac{1}{2}(25)$$

$$\int_{-2}^{5} |x|\,dx = \frac{29}{2}.$$