Homework 6; Due Tuesday, 04/25/2017

Quick Answer Questions. No work needed. No partial credit available.

Question 1. (*Fill in the blanks and multiple choice questions*)

(I) Let $G(x) = \int_x^3 e^{t^2} dt$, then fill in the blanks:

$G(x)$ is the _____ of $g(t) =$ _____ that takes the value _____ at $x =$ _____.

(II) The antiderivative $F(x)$ of $f(x) = x^2 - e^x - 1$ that satisfies $F(-1) = e$ is (*choose the correct answer(s)*)

   (a) $F(x) = x^3/3 - e^x - x$

   (b) $F(x) = 2x - xe^{x-1} + (e + 2 - e^{-2})$

   (c) $F(x) = 2x - e^x + C$

   (d) $F(x) = x^3/3 - e^x - x + (e + 1/e - 2/3)$

   (e) none of the above

(III) The antiderivative $F(x)$ of $f(x) = -2/x^3$ that satisfies $F(-2) = 0$ is (*choose the correct answer(s)*)

   (a) $F(x) = 1/x^2 - 1/4$

   (b) $F(x) = 1/x^2 + C$

   (c) $\int_{-2}^x -2/t^3 dt$

   (d) $F(x) = 1/x^2$

   (e) $F(x) = \begin{cases} 1/x^2 - 1/4 & \text{if } x < 0 \\ 1/x^2 - C & \text{if } x > 0 \end{cases}$ where C is any real number.

(IV) (*Fill in the blanks*) Given $\int_1^2 f(x)dx = 2$, $\int_2^3 f(x)dx = 4$, $\int_1^2 g(x)dx = 1$, and $\int_2^3 g(x)dx = -1$,

   (a) $\int_1^3 f(x)dx =$ _____.

   (b) $\int_1^3 g(x)dx =$ _____.

   (c) $\int_1^3 [5f(x) - 2g(x)]dx =$ _____.

Longer Questions. Provide complete justifications for your responses.

Question 2. In the following exercises, find the integrand's domain, then compute the indefinite integral. Use differentiation to justify your answers.

(a) $\int (x + e^x)dx$.

(b) $\int (-t^3 + 1)dt$.

(c) $\int \left(2t + \dfrac{1}{t\sqrt{t}}\right)(t^2 + t)dt$.

Question 3. A diver jumps from a cliff with an upward initial velocity. The cliff is 60 feet above the sea.

(a) What is the diver's initial velocity if he reaches the maximal height after 0.25 second?

(b) What is the maximal height reached by the diver?

(c) After how many seconds does the diver hit the water?

(d) What is the diver's velocity at impact?

Question 4. In the following exercises, compute the definite integral $\int_a^b f(x)dx$. At each step, write down the properties of definite integrals used.

(a) $f(x) = (x^2 - 1)(x^4 + x^3), \quad a = 1, b = 2$

(b) $f(x) = 2|x| + 1, \quad a = -1, b = 4$

(c) $f(x) = \sqrt{x}(x^2 - 1), \quad a = 0, b = 1$

Question 5. In the following exercises, compute the given integral. Use differentiation to justify your answers.
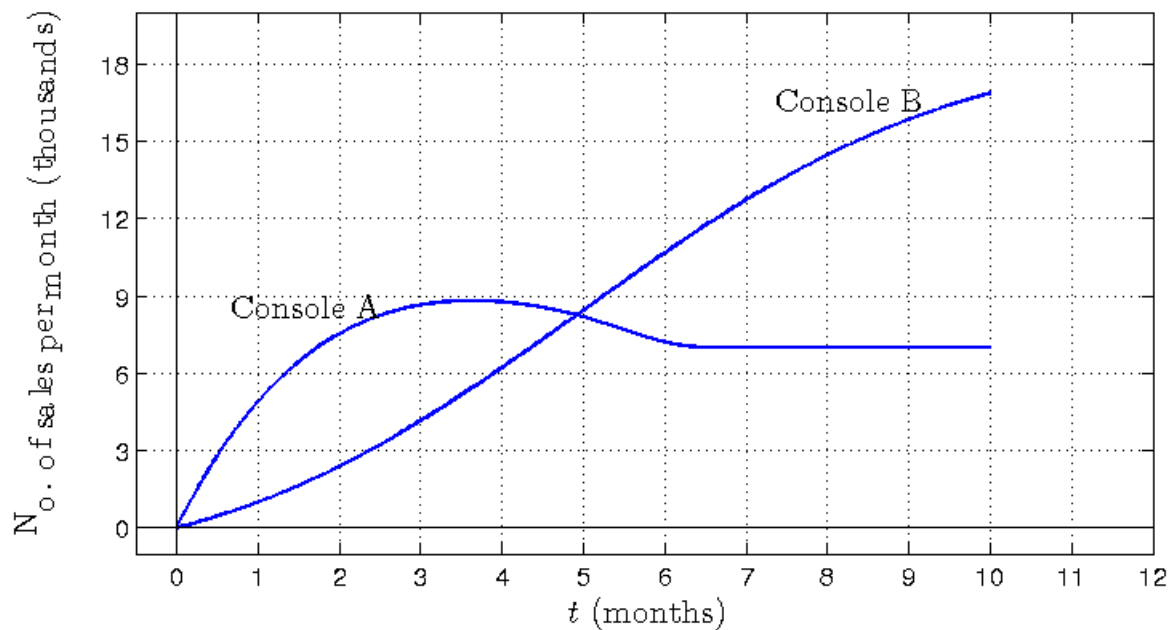
(a) $\int x(x^2 + 5)\sqrt{x^2 + 5}\,dx$.

(b) $\int (x + 1)e^{x^2 + 2x - 2}\,dx$.

(c) $\int_0^1 \dfrac{\sqrt{x}}{(2 + \sqrt{x^3})^2}\,dx$.

Question 6. The figure below shows the number of sales per month (in thousands) of two competing game consoles.

(a) Which console has the most total sales after 5 months? After 10 months? How did you arrive at your answer?

(b) At approximately what times (if any) have they sold roughly equal total amounts?

(c) Approximately how many total units of console A have been sold in 10 months?