# Homework 4; Due Thursday, 04/06/2017

**Question 1.** The Washington monument is 555 feet tall. The height $h$ of a coin dropped from the top of the monument is $h(t) = -16t^2 + 555$. Here $h$ is measured in feet from the ground, and $t$ is measured in seconds from the moment the coin was dropped.

(a) Sketch the graph of $h(t)$.

(b) How long will it take for the coin to hit the ground?

(c) Determine the coin's velocity when it hits the ground.

(d) Sketch the graph of the velocity $v(t)$. Interpret the graph.

(e) Determine the coin's acceleration $a(t)$ at time $t$.

**Question 2.** Compute the derivative of the given function by using derivative rules/formulas. At each step, specify the formula you applied.

(a) $f(x) = (x-1)(x+1)(x^2+1)$

(b) $g(x) = 2\sqrt{x^3 - 2x}$

(c) $h(t) = \dfrac{1}{t-1}\left(3 - \dfrac{1}{t+1}\right)$

(d) $p(x) = \sqrt[3]{x}(\sqrt{x} - 2x)$

(e) $j(\ell) = \sqrt{\dfrac{\ell+1}{\ell-1}}$

**Question 3.** Determine the equation of the tangent line to the curve $y = \dfrac{x^2 - 3x + 4}{4x^3 - 2x - 1}$ at the point corresponding to $x = 1$.

**Question 4.** Use the chain rule to compute the derivative of the given function.

(a) $f(x) = \left(\dfrac{4-2x}{x^2+x}\right)^4$

(b) $g(t) = \sqrt[7]{t^{10} + t^9}$

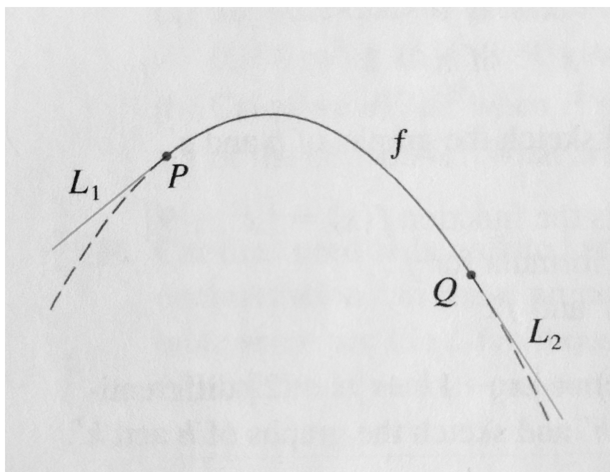
(c) $h(\ell) = \dfrac{3}{(2\ell^4 + \ell)^{99}}$

**Question 5.** Compute the value of $(f \circ g)'(t)$ at the given value of $t$.

(a) $f(y) = y^3 - 5y^2 + 1, \quad g(t) = \sqrt{t-1}, \quad t = 5$

(b) $f(y) = \dfrac{y+1}{y^2+1}, \quad g(t) = \dfrac{1}{t}, \quad t = -1$

**Question 6. Calculus@Six Flags**: Suppose you are asked to design the first ascent and drop for a new roller coaster. By studying photographs of your favorite coasters, you decide to make the slope of the ascent 0.8 and the slope of the drop $-1.6$. You decide to connect these two straight stretches $y = L_1(x)$ and $y = L_2(x)$ with part of a parabola $y = f(x) = ax^2 + bx + c$, where $x$ and $f(x)$ are measured in feet. For the track to be smooth, there can't be abrupt changes in direction, so you want the linear segments $L_1$ and $L_2$ to be tangent to the parabola at the transition points $P$ and $Q$. To simplify the equations, you decide to place the origin at $P$.




(1) Answer the following questions:

  (a) Suppose the horizontal distance between $P$ and $Q$ is 100 ft. Write equations in $a, b$ and $c$ that will ensure the track is smooth at the transition points.

  (b) Solve the equations in part (a) for $a, b$ and $c$ to find a formula for $f(x)$.

  (c) Plot $L_1, f$ and $L_2$ to verify graphically the transitions are smooth.

  (d) Find the difference in elevation between $P$ and $Q$.

(2) **Bonus (+5 pts)**: The solution in Part 1 might look smooth, but it might not *feel smooth* because the piecewise defined function [consisting of $L_1(x)$ for $x < 0$, $f(x)$ for $0 \le x \le 100$, and $L_2(x)$ for $x > 100$] doesn't have a continuous second derivative. So you decide to improve the design by using a quadratic function $q(x) = ax^2 + bx + c$ only on the interval $10 \le x \le 90$ and connecting it to the linear functions by means of two cubic functions:

$$g(x) = kx^3 + \ell x^2 + mx + n, \quad 0 \le x < 10,$$

$$h(x) = px^3 + qx^2 + rx + s, \quad 90 < x \le 100.$$

(a) Write a system of equations in 11 unknowns that ensure that the functions and their first two derivatives agree at the transition points.

(b) Solve the equations in part (a) with a computer algebra system to find formulas $q(x), g(x)$ and $h(x)$.

(c) Plot $L_1, g, q, h,$ and $L_2$, and compare the plot in Problem 1(c).