# Homework 2; Due Tuesday, 02/14/2017

**Question 1.** In the following exercises, decide whether the geometric series converges or diverges. Justify your answer. If the series converges, compute its sum.

(a) $\frac{5}{4}+\frac{5}{8}+\frac{5}{16}+\frac{5}{32}+\dots$

(b) $1-\frac{3}{2}+\frac{9}{4}-\frac{27}{8}+\cdots+\left(-\frac{3}{2}\right)^{n-1}+\dots$

(c) $\frac{1}{2}-\frac{1}{2}+\frac{1}{2}-\frac{1}{2}+\dots$

(d) $-\frac{1}{4}+\frac{1}{16}-\frac{1}{64}+\frac{1}{256}+\dots$

**Question 2.** Compute the following sums without using a calculator.

(a) $S=198+194+190+\cdots+66$

(b) $S=6561-2187+729+\cdots-3$

**Question 3.** If $\{y_n\}_n$ is a **geometric** sequence, compute the sum

$$\frac{\sqrt{y_2}}{\sqrt{y_2}-\sqrt{y_1}}+\frac{\sqrt{y_3}}{\sqrt{y_3}-\sqrt{y_2}}+\cdots+\frac{\sqrt{y_{n+1}}}{\sqrt{y_{n+1}}-\sqrt{y_n}}$$

in terms of $n$ and the common ratio $r$. All terms in the sum are well defined.

**Question 4.** Compute the limit

$$\lim_{n\to\infty}\frac{1+\frac{1}{2}+\cdots+\frac{1}{2^n}}{1+\frac{1}{3}+\cdots+\frac{1}{3^n}}.$$

**Question 5.** In this exercise, you are going to determine the shaded area enclosed by a parabola and a straight line as shown in Figure 1. The equation of the parabola is $y=-\left(\frac{2}{3}x-\frac{10}{3}\right)^2+5$. The equation of the straight line is $y=1$. Proceed by following the steps detailed below:

(a) Your first approximation, $s_1$, of the area will be the area of the (isoceles) triangle $ABC$ in Figure 2. Can you determine the area of this triangle?

*Note*: you should be easily able to find the height and base of this triangle by looking at the graph.

Is the area of this triangle less than or greater than the shaded area in Figure 1?

(b) You will now improve this approximation by adding the areas of triangles $ACD_1$ and $BCD_2$ (see Figure 3). You can use the fact that the height of each of these triangles is 0.6 units.

(i) Write down the areas of triangles $ACD_1$ and $BCD_2$.

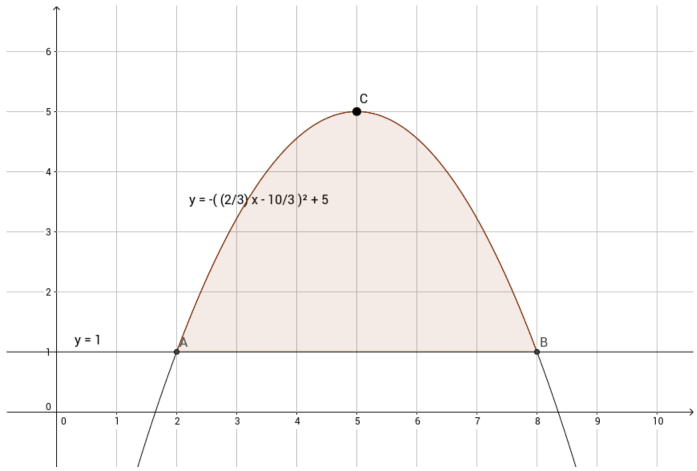


Figure 1: Area enclosed by a parabola and a straight line

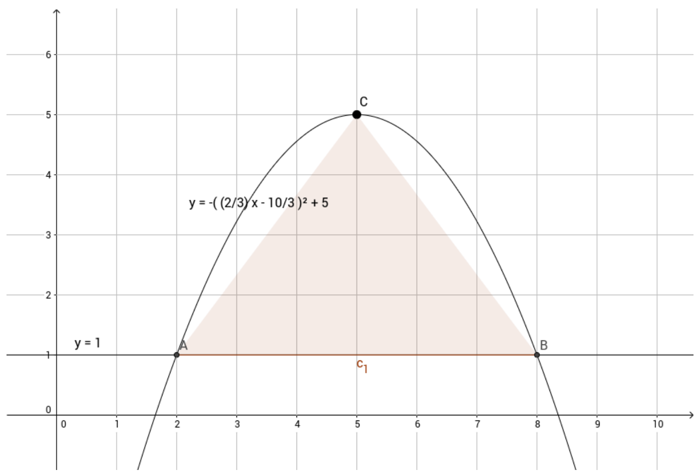


Figure 2: Approximating the area between the parabola and the straight line by the area of triangle $ABC$

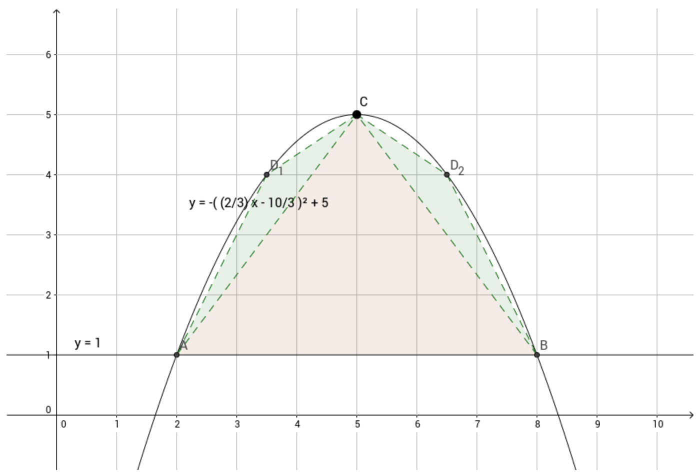


Figure 3: Improving the area estimate by considering more triangles

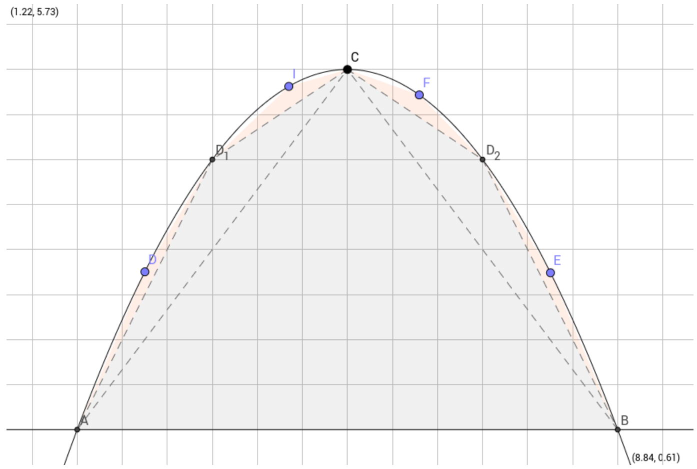


Figure 4: Improving the area estimate by considering even more triangles

(ii) What is the ratio of the area of triangle $ACD_1$ (or $BCD_2$) to the area of triangle $ABC$?

(iii) Now write down the sum of the areas of all triangles in Figure 3. i.e.,

$$s_2 = \text{area}(\Delta\text{ABC}) + \text{area}(\Delta\text{ACD}_1) + \text{area}(\Delta\text{BCD}_2).$$

(c) Now consider the area approximation as you add more and more triangles.

(i) For example, consider adding the areas of triangles $AD_1D$, $D_1CI$, $CD_2F$ and $D_2BE$ in Figure 4 to the estimate $s_2$ in step (ii).

You can assume that the area of each of these triangles is $\frac{1}{8}^{th}$ the area of triangle $ACD_1$. Using this, write down the sum of the areas of all triangles in Figure 4. i.e.,

$$\begin{aligned} s_3 = (\text{area}(\Delta\text{ABC})) + \\ (\text{area}(\Delta\text{ACD}_1) + \text{area}(\Delta\text{BCD}_2)) + \\ (\text{area}(\Delta\text{AD}_1\text{D}) + \text{area}(\Delta\text{D}_1\text{CI}) + \text{area}(\Delta\text{CD}_2\text{F}) + \text{area}(\Delta\text{D}_2\text{BE}))\,. \end{aligned}$$

(ii) What happens if you add an infinite number of triangles in the above fashion? Can you write down the area estimate as an infinite series?

(iii) If you wrote down a series in part (ii), can you identify the type of series? Does the series converge? If yes, what is the sum?