Mid-Term Exam Review – MTH 305 (Spring 2017)

(1) Determine direct and recursive formulas for the sequences whose first few terms coincide with the given ones:

(a) $6, -12, -30, -48, \ldots$

(b) $2, -4, 8, -16, \ldots$

(2) Determine a recursive formula for the sequence whose first few terms coincide with the given ones:

(a) $3, 6, 9, 15, 24, 39, 63 \ldots$

(b) $6, 18, 54, 162, \ldots$

(3) Determine direct and recursive formulas for the given sequences below. Give the value of the sequence's first term.

(a) An arithmetic sequence $\{x_n\}_{n\geq 1}$ with $x_3 = 4$ and a common difference of $\dfrac{1}{3}$.

(b) A geometric sequence $\{x_n\}_{n\geq 1}$ with $x_2 = 4$ and a common ratio of $-\dfrac{1}{4}$.

(c) An arithmetic sequence $\{x_n\}_{n\geq 1}$ with $x_8 = 40$ and $x_{20} = -20$.

(d) An geometric sequence $\{x_n\}_{n\geq 1}$ with $x_5 = 64$ and $x_2 = 512$.

(4) For each of the following, determine whether or not they converge. If they converge, what is the limit? Provide some algebraic justification.

(a) $\left\{\cos\left(\dfrac{n\pi}{2}\right)\right\}_{n\in\mathbb{N}}$

(b) $\{(-1)^n n\}_{n\in\mathbb{N}}$

(c) $\left\{\sqrt{n^2+3} - n\right\}_{n\in\mathbb{N}}$

(d) $\left\{-1 + \dfrac{(-1)^n}{n}\right\}_{n\in\mathbb{N}}$

(5) Determine if the following sequences are bounded. Briefly justify your answers.

(a) $\{(-1)^n(2n+1) - 3\}_{n\in\mathbb{N}}$

(b) $\left\{\dfrac{n}{n+5}\right\}_{n\in\mathbb{N}}$

(6) For each of the following, determine whether or not the given sequence is convergent or divergent. If convergent, determine its limit. Use the properties of limits discussed in class to justify your answer.

(a) $\{(-1)^n + 3\}_{n\in\mathbb{N}}$

(b) $\left\{\dfrac{5n}{n+3}\right\}_{n\in\mathbb{N}}$

(c) $\left\{\dfrac{2-(-1)^n}{n}\right\}_{n\in\mathbb{N}}$

(7) Decide whether the geometric series converge or diverge. Justify your answer. If the series converges, compute its sum.

(a) $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \dots$

(b) $3 - 3 + 3 - 3 + 3 - 3 \cdots +$

(c) $\sum_{n=1}^{\infty} \frac{7}{8^{n-1}}$

(d) $\frac{3}{4} + \frac{9}{16} + \cdots + \left(\frac{3}{4}\right)^n + \dots$

(8) (*Writing recurring decimals as fractions*) Let $x = 0.\overline{35}$. Show that $x = \frac{35}{99}$. (note: the notation $0.\overline{35}$ means that the digits 3 and 5 repeat indefinitely; i.e., $x = 0.353535353535\dots$. Hint: try and write x as a geometric series!)

(9) Compute the sum without using a calculator.

(a) $S = 6 - 3 + \frac{3}{2} - \frac{3}{4} + \cdots - \frac{3}{64}$

(b) $S = 3 + 8 + 13 + \cdots + 48$

(10) Determine if the given equation describes y as a function of x.

(a) $y = -3$

(b) $x = 2y^2 + 3$

(c) $y^2 = 2x^2 - 1$

(d) $y = \sqrt{x - 4}$

(e) $2y^3 + 5x = 10$

(f) $y = \begin{cases} x + 2 & \text{if } x < 0, \\ 1 - x & \text{if } x > 0. \end{cases}$

(11) Determine the domain of the given function.

(a) $f(x) = \frac{1}{\sqrt[2]{x^2 - 8x}}$

(b) $g(x) = \frac{x - 3}{x^2 - 16}$

(c) $h(t) = \sqrt[3]{3t - 2}$

(12) Determine the assignment rule and the domain of the specified composite functions.

(a) $f \circ g$ if $f(x) = x + 3$ and $g(x) = \frac{1}{x}$.

(b) $g \circ f$ if $f(x) = \frac{x}{x^2 + 1}$ and $g(x) = \sqrt{x}$.

(c) $f \circ g$ and $g \circ f$ if $f(x) = x + \frac{1}{x}$ and $g(x) = \frac{x + 1}{x + 2}$.

(13) Sketch the graph of the function and use it to determine the values of a for which $\lim_{x\to a} f(x)$ exists.

$$f(x) = \begin{cases} 1+\sin x & \text{if } x < 0 \\ \cos x & \text{if } 0 \le x \le \pi \\ \sin x & \text{if } x > \pi \end{cases}$$

(14) Sketch the graph of an example of a function f that satisfies all of the given conditions.

(a) $\lim_{x\to 0^-} f(x) = 2, \quad \lim_{x\to 0^+} f(x) = 0, \quad \lim_{x\to 4^-} f(x) = 3, \quad \lim_{x\to 4^+} f(x) = 0 \quad f(0) = 2, \quad f(4) = 1.$

(15) Let

$$g(x) = \begin{cases} x & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 2 - x^2 & \text{if } 1 < x \le 2 \\ x - 3 & \text{if } x > 2 \end{cases}$$

(a) Evaluate each of the following, if it exists.

(i) $\lim_{x\to 1^-} g(x)$

(ii) $\lim_{x\to 1} g(x)$

(iii) $g(1)$

(iv) $\lim_{x\to 2^-} g(x)$

(v) $\lim_{x\to 2^+} g(x)$

(vi) $\lim_{x\to 2} g(x)$

(b) Sketch the graph of g.

(16) Evaluate the limit, if it exists

(a) $\lim_{x\to -3} \dfrac{\sqrt{x^2+16}-5}{x+3}$

(b) $\lim_{x\to 3} \dfrac{x^2-x-6}{x-3}$

(c) $\lim_{h\to 0} \dfrac{(x+h)^2-x^2}{h}$

(17) Look over HW2 problem 5.