Name: _____

READ THE FOLLOWING INSTRUCTIONS.

- **Do not open your exam until told to do so.**
- No calculators, cell phones or any other electronic devices can be used on this exam.
- Clear your desk of everything except pens, pencils and erasers.
- Without fully opening the exam, check that you have pages 1 through 20.
- Fill in your name, etc. on this first page.
- **Show all your work.** Write your answers clearly! Include enough steps for the grader to be able to follow your work. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand and a proctor will come to you.
- There is no talking allowed during the exam.
- You will be given exactly 80 minutes for this exam.

ACADEMIC HONOR CODE

As a student and citizen of the Michigan State University Community I pledge to not lie, cheat, or steal in my academic endeavors.

I have read and understand the above instructions: _____

SIGNATURE

Quick Answer Questions. No partial credit available; No justification necessary.

1. Fill in the blanks below.

   (a) (2 points) Determine if the following sequences are bounded.

   (i) $\left\{(-1)^n(2n^2-3)-27\right\}_{n\in\mathbb{N}}$ _____.

   (ii) $\left\{\dfrac{3n}{n+8}\right\}_{n\in\mathbb{N}}$ _____.

   (b) (4 points) Determine if the given equation describes y as a function of x.

   (i) $x = -2y^2 + 6$ _____

   (ii) $y^2 = 7x^2 - 6$ _____

   (iii) $y = \sqrt{x-16}$ _____

   (iv) $2y^5 - 7x = 8$ _____

   (c) (4 points)

$$\text{Let} \qquad g(x) = \begin{cases} x & \text{if} \quad x < 1 \\ 3 & \text{if} \quad x = 1 \\ 2 - x^2 & \text{if} \quad 1 < x \leq 2 \\ x - 3 & \text{if} \quad x > 2 \end{cases}$$

   Evaluate each of the following, if it exists.

   1. $\lim\limits_{x\to 1^-} g(x)$ _____

   2. $\lim\limits_{x\to 1} g(x)$ _____

   3. $\lim\limits_{x\to 2^+} g(x)$ _____

   4. $\lim\limits_{x\to 2} g(x)$ _____

Additional Work Space:

Complete Explanation Questions. Provide complete justifications for your responses.

2. Determine direct and recursive formulas for the given sequences below. Give the value of the sequence's first term.

   (a) (5 points) An arithmetic sequence $\{x_n\}_{n\geq 1}$ with $x_3 = 0$ and $x_7 = 40$.

   (b) (5 points) A geometric sequence $\{y_n\}_{n\geq 1}$ with $y_3 = -6$ and $y_6 = \dfrac{2}{9}$.

Additional Work Space:

3. For each of the following, determine whether or not they converge. If they converge, what is the limit? Provide some algebraic justification.

   (a) (5 points) $\left\{\dfrac{(-1)^n}{n^2}+\dfrac{3}{4}\right\}_{n\in\mathbb{N}}$

   (b) (6 points) $\left\{n-\sqrt{n^2+7}\right\}_{n\in\mathbb{N}}$

Additional Work Space:

4. (8 points) Using the properties of limits, find the limit of the following convergent sequence

$$\left\{ \frac{4n}{n+7} \right\}_{n \in \mathbb{N}}.$$

5. Decide whether the geometric series below converge or diverge. Justify your answer. If the series converges, compute its sum.

   (a) (5 points) $2 - \frac{2}{3} + \frac{2}{9} - \frac{2}{27} + \dots$

   (b) (5 points) $\sum_{n=1}^{\infty} \frac{3}{7^{n-1}}$

Additional Work Space:

6. (8 points) Compute the sum $S = -\frac{1}{4} + \frac{1}{2} - 1 + \cdots + 128$ without using a calculator.

7. (8 points) Let $x = 0.\overline{75}$. Show that $x = \dfrac{75}{99}$. (note: the notation $0.\overline{75}$ means that the digits 7 and 5 repeat indefinitely; i.e., $x = 0.757575757575\ldots$.

8. (10 points) Find the assignment rule (or law/formula) and the domain of the specified composite functions below:

$$f \circ g \quad \text{and} \quad g \circ f, \quad \text{if} \quad f(x) = 1 - \frac{1}{x} \quad \text{and} \quad g(x) = \frac{x}{x+3}.$$

Additional Work Space:

9. Evaluate the following limits, if they exist. Show the steps used and use limit properties to justify your answer.

   (a) (5 points) $\lim_{x \to 5} \dfrac{x^2 - x - 20}{x - 5}$

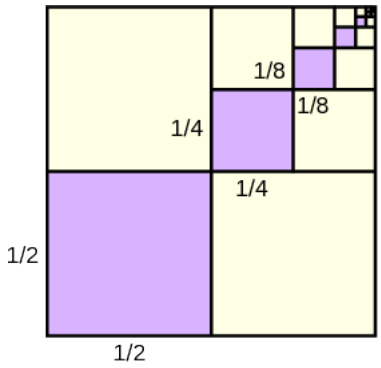
   (b) (6 points) $\lim_{x \to -1} \dfrac{\sqrt{x^2 + 3} - 2}{x + 1}$

Additional Work Space:

10. (6 points) Sketch the graph of an example of a function f that satisfies all of the given conditions.

$$\lim_{x\to 0^-} f(x) = 3, \quad \lim_{x\to 0^+} f(x) = -1, \quad \lim_{x\to 4^-} f(x) = 3, \quad \lim_{x\to 4^+} f(x) = 2 \quad f(0) = -1, \quad f(4) = 0.$$

11. (8 points) Find the sum of the areas of the shaded squares in the figure below. (Note: you may assume that there are an infinite number of shaded squares following the same pattern.)



Additional Work Space:

Congratulations you are now done with the exam!
Go back and check your solutions for accuracy and clarity.

When you are completely happy with your work please bring your exam to the front to be handed in.
Please have your MSU student ID ready so that is can be checked.

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
2	10	
4	10	
6	11	
8	8	
9	10	
11	8	
12	8	
13	10	
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18	8	
Total:	100	