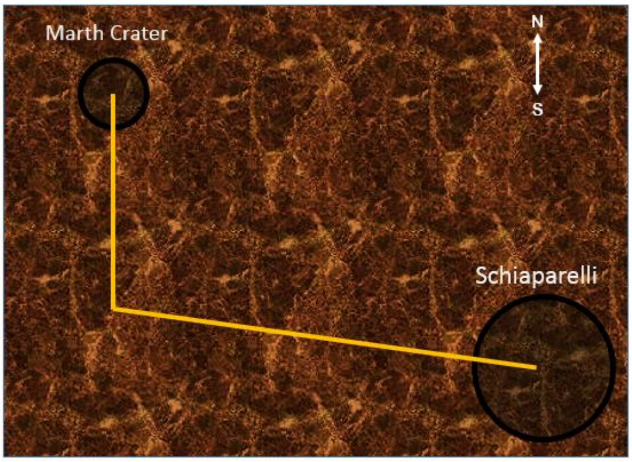
Bring Him Home!

Background

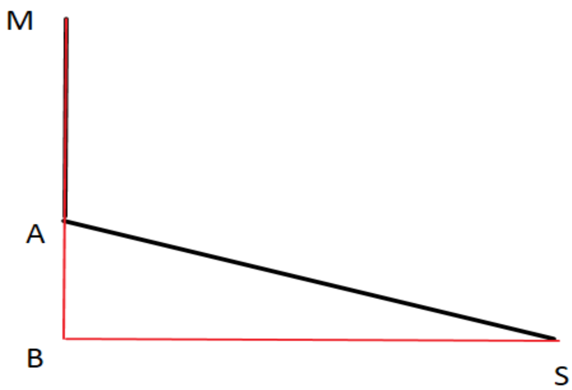
As a mathematician working for NASA you have been called in to help botanist and astronaut Mark Watney who is stranded on Mars. Your skills are needed to bring him home.

Mission

Mark Watney is currently at Marth crater. His only hope of survival is if he can make it to the MAV rocket which is located at the centre of the Schiaparelli crater. Assume his solar-powered Rover craft can travel at 3.75 km/h but is currently running at A% efficiency due to a slow moving dust storm between Marth and Schiaparelli. Mark has just started travelling south. At some point he needs to turn left and head for Schiaparelli but whenever he does this the dust storm will decrease the Rover's efficiency to B%. One possible path is shown below.



a) Using the NASA Mars Trek software, measure how far south Schiaparelli is from Marth and how far east Schiaparelli is from Marth. These distances are from M to B and from B to S respectively on the diagram opposite. Use the Tool option in the Mars Trek software to achieve this.



b) Now you need to find how far south Mark should travel before turning left so that he will get to Schiaparelli in the minimum time. This will be the distance from M to A on the diagram. This distance is not drawn to scale on the diagram. Assume Mark is travelling across level ground. Construct a spreadsheet with the following headings and use the spreadsheet to find how far south Mark should travel before turning.

	A	B	C	D	E	F	G
1	M to A (km)	A to B (km)	B to S (km)	A to S (km)	Time M to A (hrs)	Time A to S (hrs)	Total time (hrs)

c) Use the NASA Mars Trek software to produce an image of the surface of Mars with the path that Mark should take highlighted on it in yellow.

(based on exercises by Dr. Ian Thomson, Ormiston College)

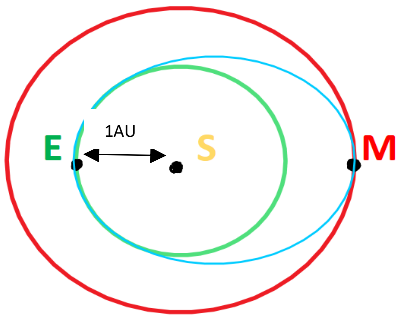
Mission 2

Meanwhile back on Earth, Rich Purnell, an astrophysicist, has the idea of using the gravitational force of Earth to assist the Hermes return to Mars – here is a youtube clip from the recent "The Martian" film, capturing Mr Purnell's idea.

In astronomy, Kepler's laws of planetary motion are three scientific laws describing the motion of planets around the Sun. According to Kepler's laws:

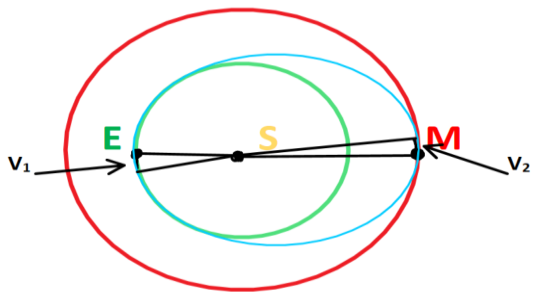
1. The orbit of a planet is an ellipse with the Sun at one of the two foci. The diagram below (see Figure 1) shows Earth's orbit in green and Mars' orbit in red. Click here for some background information on the properties and features of an ellipse.
2. A line segment b/w a planet and the Sun sweeps out equal areas during equal intervals of time
3. The square of the orbital period of a planet (T) is proportional to the cube of the semi-major axis (a) of its orbit. This can be described by the equation $T^2 = a^3$, where T is in units of Earth years (so one complete rotation around the Sun for Earth = 1 Earth year), and a is in astronomical units (AU). One AU is the straight line distance from Earth to the Sun which is approximately 149 598 000 km.

For the Hermes to travel from Earth to Mars and back again, it would travel an elliptical orbit around the Sun as shown in blue in the diagram opposite. The major axis of this ellipse would be a straight line from the Earth through the Sun to Mars. The distance from Earth to the Sun is 1 AU, and the distance from Mars to the Sun is approximately 1.523691 AU.



a) Use Kepler's 3rd Law, find the time, T of Hermes' orbit (i.e. one complete rotation of the blue ellipse).
b) How long will it take the Hermes to get to Mars?
c) The Hermes and Mars have to arrive together at the point M. Where should Mars be on its orbit around the Sun when the Hermes leaves Earth?

Let the speed of the Hermes in km/s at Earth be V_1 and at Mars be V_2. According to Kepler's 2nd Law the two long thin triangles shown in the diagram opposite will have equal area. In a time interval of one second, the length of the base of these triangles will be V_1 and V_2 respectively



d) By applying Kepler's 2nd Law, show that $r_e V_1 = r_m V_2$ where r_e is the radius of the Earth's circular orbit around the Sun and r_m is the radius of Mars' circular orbit around the Sun.

The energy E of the Hermes on leaving Earth will be the same as its energy on arrival at Mars. This is described in the equation below:

$\frac{1}{2} m V_1^{\ 2} - \frac{km}{r_e} = \frac{1}{2} m V_2^{\ 2} - \frac{km}{r_m}$, where m is mass, k =1.32818 x 10^{11}, V_1 is the speed of the Hermes as it leaves Earth, and V_2 is its speed on arrival at Mars.

e) Show that $V_1^{\ 2} - V_2^{\ 2} = 2k\left(\frac{1}{r_e} - \frac{1}{r_m}\right)$
f) Find V_1, the speed of the Hermes when it leaves Earth [Hint: Use answer from part (a) above].

(based on exercises by Dr. Ian Thomson, Ormiston College)