Comments on Quiz 3 Q#1

Let $f(x) = \frac{x^2-x-2}{x+1}$

Note: $x^2-x-2 = (x-2)(x+1)$

* Is $f(x)$ defined at $x=-1$?

to see if $f(x)$ is defined at some $x=a$, directly plug in $x=a$ in the rule/formula.

$f(x)$ is not defined at $x=-1$ (since the denominator goes to 0)

no simplifications/factorizations/cancellations allowed

* Consider $\lim_{x\to -1} f(x) = \lim_{x\to -1} \frac{(x^2-x-2)}{x+1} = \lim_{x\to -1} \frac{(x-2)\cancel{(x+1)}}{\cancel{(x+1)}} = \lim_{x\to -1} (x-2) = -3$

limit exists and $\lim_{x\to -1} f(x) = -3$.

here, factorization/cancellation allowed since limits are only concerned with behavior of f at x close to a, and not at $x=a$.

* Since $f(-1)$ is not defined, f is not continuous at $x=-1$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

Chain Rule

If g is differentiable at x and f is differentiable at $g(x)$, then the composite function $F = f \circ g$ defined by $F(x) = f(g(x))$ is differentiable at x and

$$F'(x) = f'(g(x)) \cdot g'(x)$$

(alternate notation) If $y = f(u)$ and $u = g(x)$ then $$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Example: If $F(x) = \sqrt{x^3+1}$, compute $F'(x)$.

We have $F(x) = (f \circ g)(x) = f(g(x))$, where $f(u) = \sqrt{u}$ and $g(x) = x^3+1$.

We know that $f'(u) = \dfrac{1}{2\sqrt{u}}$ and $g'(x) = 3x^2$

By the chain rule, we have

$$F'(x) = f'(g(x)) \cdot g'(x)$$

$$= \frac{1}{2\sqrt{g(x)}} \cdot 3x^2$$

$$= \frac{1}{2\sqrt{x^3+1}} \cdot 3x^2$$

$$\boxed{F'(x) = \frac{3x^2}{2\sqrt{x^3+1}}}$$

(#3) Compute the derivative of

(e) $h(t) = 3\sqrt{3t^2+2t+1}$

Using derivative rule, we have

$$\frac{dh}{dt} = 3\,\frac{d}{dt}\left(\sqrt{3t^2+2t+1}\right) \qquad \text{(constant multiple rule)}$$

$$= 3\,\frac{\frac{d}{dt}(3t^2+2t+1)}{2\sqrt{3t^2+2t+1}} \qquad \text{(square root rule)}$$

$$= \frac{3\left(3\,\frac{d}{dt}(t^2) + 2\,\frac{d}{dt}(t) + \frac{d}{dt}(1)\right)}{2\sqrt{3t^2+2t+1}} \qquad \text{(sum, constant mult. rules)}$$

$$= \frac{3(6t+2+0)}{2\sqrt{3t^2+2t+1}} \qquad \text{(power rule)}$$

$$= \frac{18t+6}{2\sqrt{3t^2+2t+1}}, \quad \text{or} \quad \boxed{h'(t) = \frac{9t+3}{\sqrt{3t^2+2t+1}}}$$