Differentiation Formulas

Rule	Function	Derivative
Power Rule	x^a, a rational	$a x^{a-1}$
Constant multiple rule	$c f(x)$, c real	$c f'(x)$
Sum rule	$f(x) + g(x)$	$f'(x) + g'(x)$
Reciprocal rule	$\frac{1}{f(x)}$	$-\frac{f'(x)}{f^2(x)}$
Product Rule	$f(x) g(x)$	$f'(x) g(x) + f(x) g'(x)$
Quotient rule	$\frac{f(x)}{g(x)}$	$\frac{f'(x) g(x) - f(x) g'(x)}{g^2(x)}$
Square root rule	$\sqrt{f(x)}$	$\frac{f'(x)}{2\sqrt{f(x)}}$

Derivatives of f: f', f'', f''', $f^{(4)}$, ... ← 4th derivative

1st derivative

2nd derivative

Note:
derivative of a constant function
$$\frac{d}{dx}(c) = 0$$

Examples:

① Compute the derivative of $f(x) = x^7 + 6x^5 - \frac{2}{3}x^2 + \sqrt{3}$.

We have $f'(x) = \frac{d}{dx}\left(x^7 + 6x^5 - \frac{2}{3}x^2 + \sqrt{3}\right)$

$= \frac{d}{dx}(x^7) + \frac{d}{dx}(6x^5) + \frac{d}{dx}\left(-\frac{2}{3}x^2\right) + \frac{d}{dx}(\sqrt{3})$; (Sum rule)

$= \frac{d}{dx}(x^7) + 6\frac{d}{dx}(x^5) - \frac{2}{3}\frac{d}{dx}(x^2) + \sqrt{3}\frac{d}{dx}(x^0)$; (Const. mult. rule)

$= 7x^6 + 6(5x^4) - \frac{2}{3}(2x) + 0$ (power rule)

$$\boxed{f'(x) = 7x^6 + 30x^4 - \frac{4}{3}x}$$

② Find $F'(x)$ if $F(x) = (3x^2)(8x^5)$

By the product rule,

$$F'(x) = (3x^2)\frac{d}{dx}(8x^5) + (8x^5)\frac{d}{dx}(3x^2)$$

$$= (3x^2)(8(5)x^4) + (8x^5)(3(2)x)$$

$$= (3x^2)(40x^4) + (8x^5)(6x)$$

$$= 120x^6 + 48x^6$$

$$F'(x) = 168x^6$$