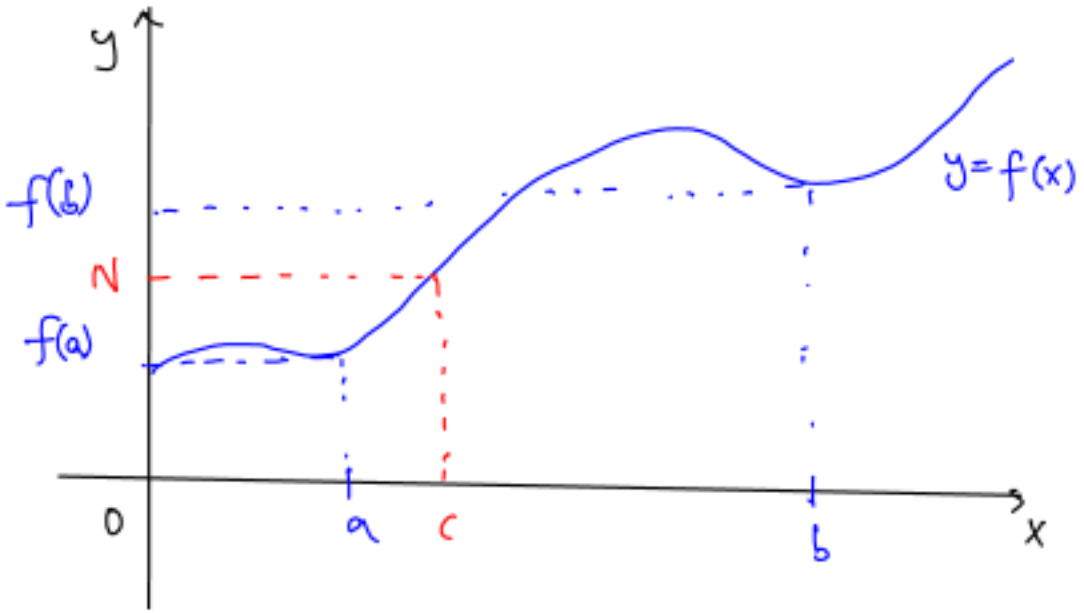
Intermediate Value Theorem

Suppose that f is continuous on the closed interval $[a, b]$ and let N is any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$.

Then there exists a number c in (a, b) such that $f(c) = N$.



(#6) Use the intermediate value theorem to show that there is a root of the equation $x^4+x-3=0$ between 1 and 2.

Let $f(x) = x^4 + x - 3$.

We note that $f(1) = 1^2 + 1 - 3 = -1$ $\quad f(1) < 0$

$f(2) = 2^4 + 2 - 3 = 15$ $\quad f(2) > 0$

Now, from the intermediate value theorem,

there exists $c \in (1,2)$ such that $f(1) < f(c) < f(2)$

Letting $f(c) = 0$ (note that $-1 < 0 < 15$ holds) implies that c is a root of the equation x^4+x-3 and that this root lies between 1 and 2.