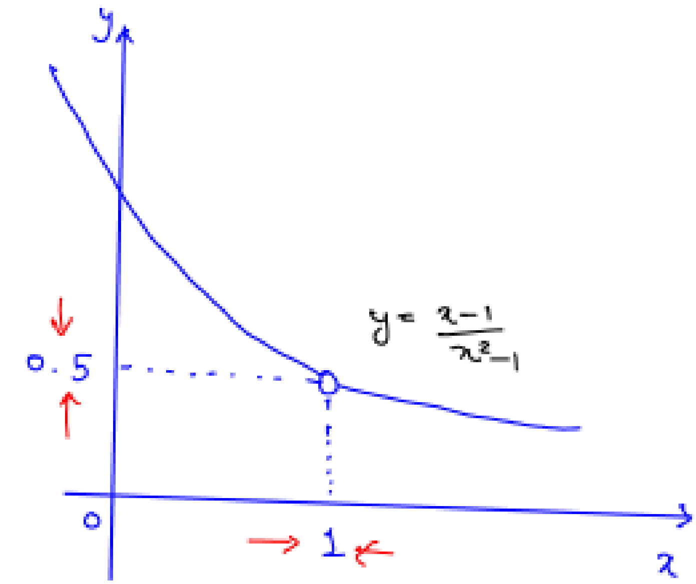
The Limit of a Function

Consider the function $f(x) = \frac{x-1}{x^2-1}$ (note that $f(x)$ is not defined when $x=1$.)

Let us investigate the behavior of f for values of x near 1.

$x<1$	$f(x)$
0.5	0.666667
0.9	0.526316
0.99	0.502513
0.999	0.500250
0.9999	0.500025

$x>1$	$f(x)$
1.5	0.400000
1.1	0.476190
1.01	0.497512
1.001	0.499750
1.0001	0.499975



as x gets closer to 1 (on either side of 1), $f(x)$ gets closer to 0.5.

We say that $\lim_{x \to 1} \frac{x-1}{x^2-1} = 0.5$

Limit Definition

Suppose $f(x)$ is defined when x is near the number a (this means that f is defined on some open interval that contains a, except possibly at a itself)

Then, we write

$$\lim_{x \to a} f(x) = L$$

and say "the limit of $f(x)$ as x approaches a, equals L"

if we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a (on either side of a) but not equal to a.

alternate notation

$f(x) \to L$ as $x \to a$

"$f(x)$ approaches L as x approaches a"

One-sided limits

We write

$$\lim_{x \to a^-} f(x) = L$$

and say that the left-hand limit of $f(x)$ as x approaches a from the left is equal to L if we can make the values of $f(x)$ arbitrarily close to L by taking x sufficiently close to a with x less than a.

Similarly, if we require that x be greater than a, we get

"the right-hand limit of $f(x)$ as x approaches a is equal to L"

and we write

$$\lim_{x \to a^+} f(x) = L$$