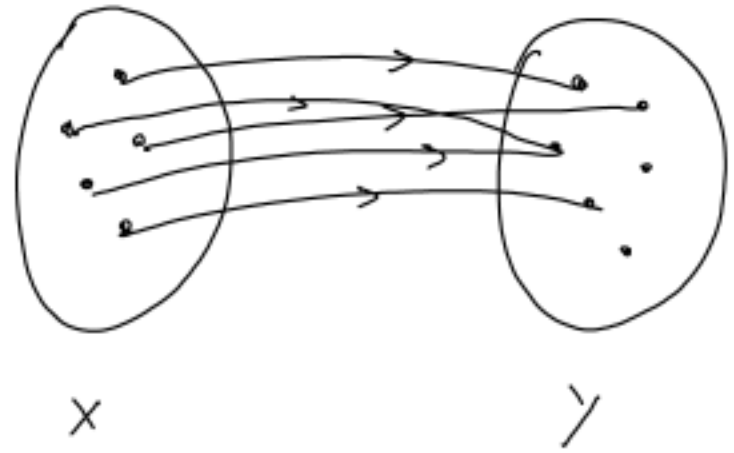
Functions

Defn Suppose X and Y are sets. A function or map from X to Y is an association between the members of the sets. More precisely, for every element of X, there is a unique element of Y.

If f is a function from X to Y, then we write $f : X \to Y$, and the unique element in Y associated to x is denoted $f(x)$.

The set X is called the domain (or source) of f and Y is called the codomain (or target) of f. the element $f(x)$ is called the value of x under f.



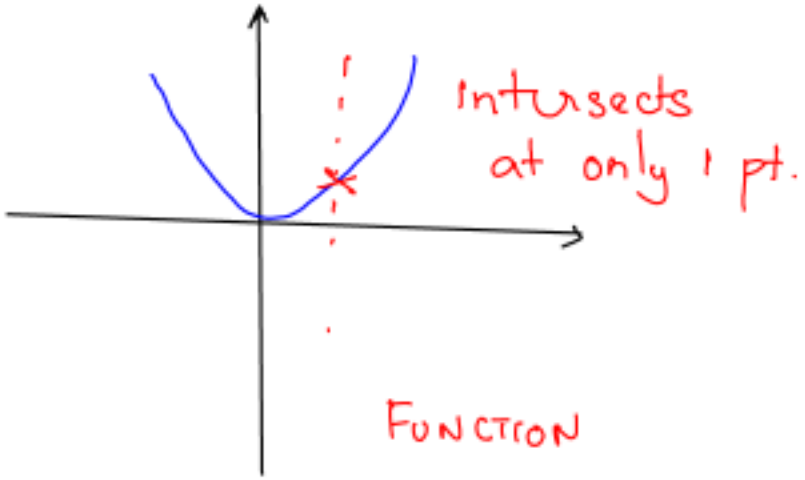
The set $\{f(x) : x \in X\}$ is called the range of f.

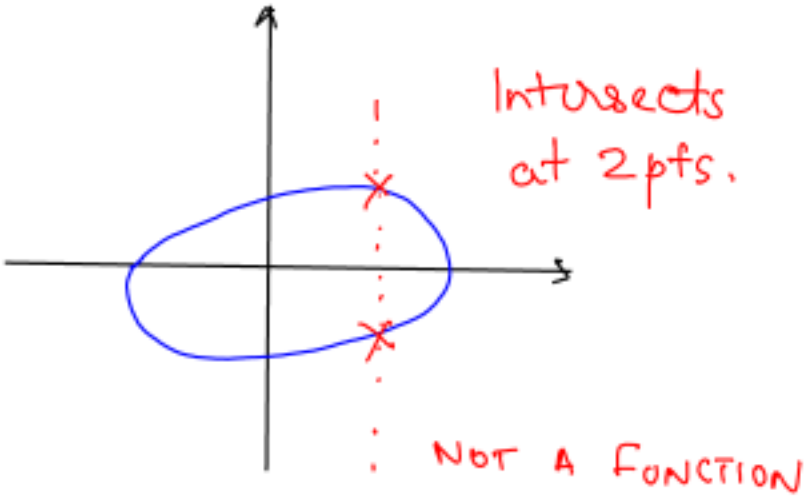
Graph of a function

The graph of a function f is the collection of all points in the plane with coordinates $(x, f(x))$ for all values of x in the domain of f.

Vertical Line Test

A curve the plane will represent the graph of a function f if and only if any vertical line intersects the curve only once.





Examples

Consider the assignment rule $f(x) = \frac{1}{x-1}$. Does this describe a function from $\mathbb{R}$ to $\mathbb{R}$?

This does not describe a function from $\mathbb{R}$ to $\mathbb{R}$ since $f(1)$ is undefined. However, let $A = \{x \in \mathbb{R} \mid x \neq 1\}$. Then $f: A \to \mathbb{R}$ is a function.

(A: domain of f)

Note:

* The function $f: \mathbb{R} \to \mathbb{R}$ with

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0,$$

where $a_0, a_1, \dots, a_n$ are real numbers and n is a non-negative integer is called a polynomial of degree n (when $a_n \neq 0$).

* the quotient of two polynomial functions is called a rational function.

Ex: $g(x) = \frac{x}{x^2-1}$ (here domain (g) is the set of all real numbers not including ± 1)

Find the largest possible set $A \subseteq \mathbb{R}$ such that $f : A \to \mathbb{R}$ defines a function.

(i) $f(x) = 1 + x^2$ (ii) $f(x) = \sqrt{3x-1}$ (iii) $f(x) = \frac{x}{x-3}$ (iv) $f(x) = \frac{x}{x^2-x-6}$

$A = \mathbb{R}$ $A = \{x \in \mathbb{R} \mid x \geq \frac{1}{3}\}$ $A = \{x \in \mathbb{R} \mid x \neq 3\}$ $A = \{x \in \mathbb{R} \mid x \neq 3 \text{ and } x \neq -2\}$