Decimal Expansions

Every positive real number x has a decimal representation

$$x = a_0 . a_1 a_2 a_3 \ldots\ldots\ldots$$

where $0 \leq a_k \leq 9$ for each $k \geq 1$.

For example, let $x = 0.44444\ldots\ldots$, or $x = 0.\overline{4}$

then, we have

$$0.4 = \frac{4}{10}$$

$$0.44 = \frac{4}{10} + \frac{4}{100}$$

$$0.444 = \frac{4}{10} + \frac{4}{100} + \frac{4}{1000}$$

$$\vdots$$

hence, $0.\overline{4} = \frac{4}{10} + \frac{4}{100} + \frac{4}{1000} + \ldots\ldots\ldots$

- this is an infinite (geometric) series with first term $a = \frac{4}{10}$ and common ratio $r = \frac{1}{10}$

- Since $|r| < 1$, we know that the series converges

and $$L = \lim_{n \to \infty} s_n = \lim_{n \to \infty} \sum_{k=1}^{n} \frac{4}{10^n}$$

$$= \frac{a}{1-r}$$

$$= \frac{4/10}{1 - 1/10}$$

$$= \frac{4/10}{9/10}$$

$$= \frac{4}{9}$$

- Question: can you guess $0.\overline{9} = ?$