LECTURE 2

1/12/2017

Types of Sequences

① **Arithmetic Sequences** A sequence is an arithmetic sequence if the next term in the sequence is obtained by adding a fixed number d to the current term.

$\{x_n\}_{n\in\mathbb{N}}$ is an arithmetic sequence provided $x_{n+1} = x_n + d$ for $n \geq 1$

here, x_1 is the first term

d is the common difference

Note: (Recursive formula) $x_{n+1} = x_n + d$

(Direct formula) $x_n = x_1 + (n-1)d$ for each $n \in \mathbb{N}$.

② **Geometric Sequence** A sequence is a geometric sequence if the next term of the sequence is found by multiplying the current term by a fixed number r.

$\{x_n\}_{n\in\mathbb{N}}$ is a geometric sequence provided $x_{n+1} = x_n \cdot r$ for $n \geq 1$

here, x_1 is the first term

r is the common ratio

Note: (recursive formula) $x_{n+1} = x_n \cdot r$ for $n \geq 1$

(direct formula) $x_n = x_1 \cdot r^{n-1}$ for $n \geq 1$.

LIMITS OF SEQUENCES

What happens to the terms of $\{x_n\}$ as n approaches infinity?

Def

Def^n: A sequence $\{x_n\}_{n\in\mathbb{N}}$ is said to have a limit L if, for every positive number ε, there is another positive number N (which may depend on ε) such that

$$|x_n - L| < \varepsilon \quad \text{for all } n \geq N.$$

In this case, we say the sequence $\{x_n\}_{n\in\mathbb{N}}$ converges to L and we write

$$\lim_{n\to\infty} x_n = L \qquad \text{or} \qquad x_n \longrightarrow L \text{ as } n \longrightarrow \infty.$$

A sequence which does not converge is called divergent.

(Alternate interpretation) $\{x_n\}_{n\in\mathbb{N}}$ converges to L provided <u>any</u> open interval centered at L contains all but finitely many terms of the sequence.

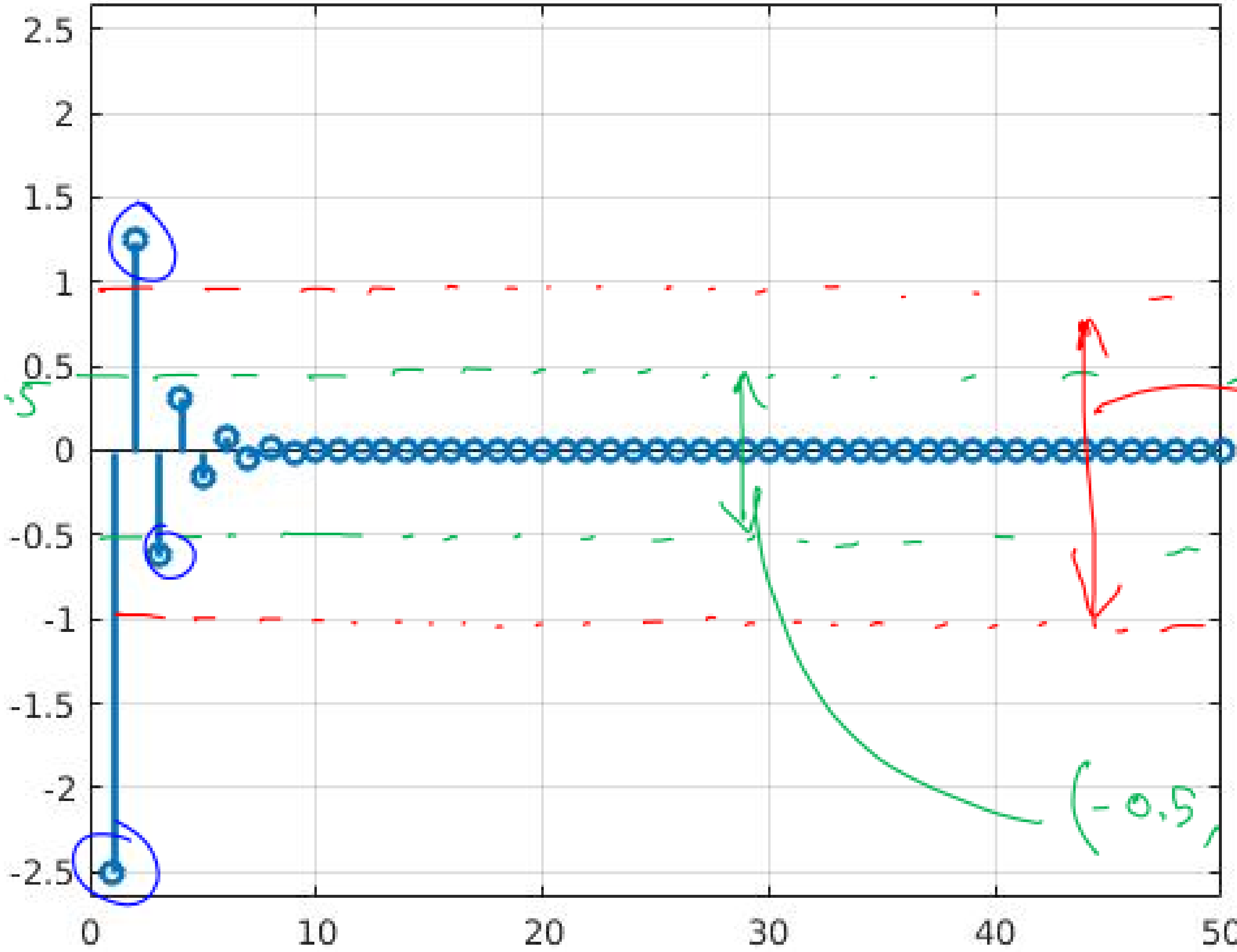
Example $\{x_n\}_{n \in \mathbb{N}}$ with $x_n = \dfrac{5(-1)^n}{2^n}$

$L = 0$

plot of first few terms

$\varepsilon = 1$

$\varepsilon = 0.5$



$(-1, 1)$

open interval centred at $L=0$

all but 2 terms are contained in this interval

$(-0.5, 0.5)$

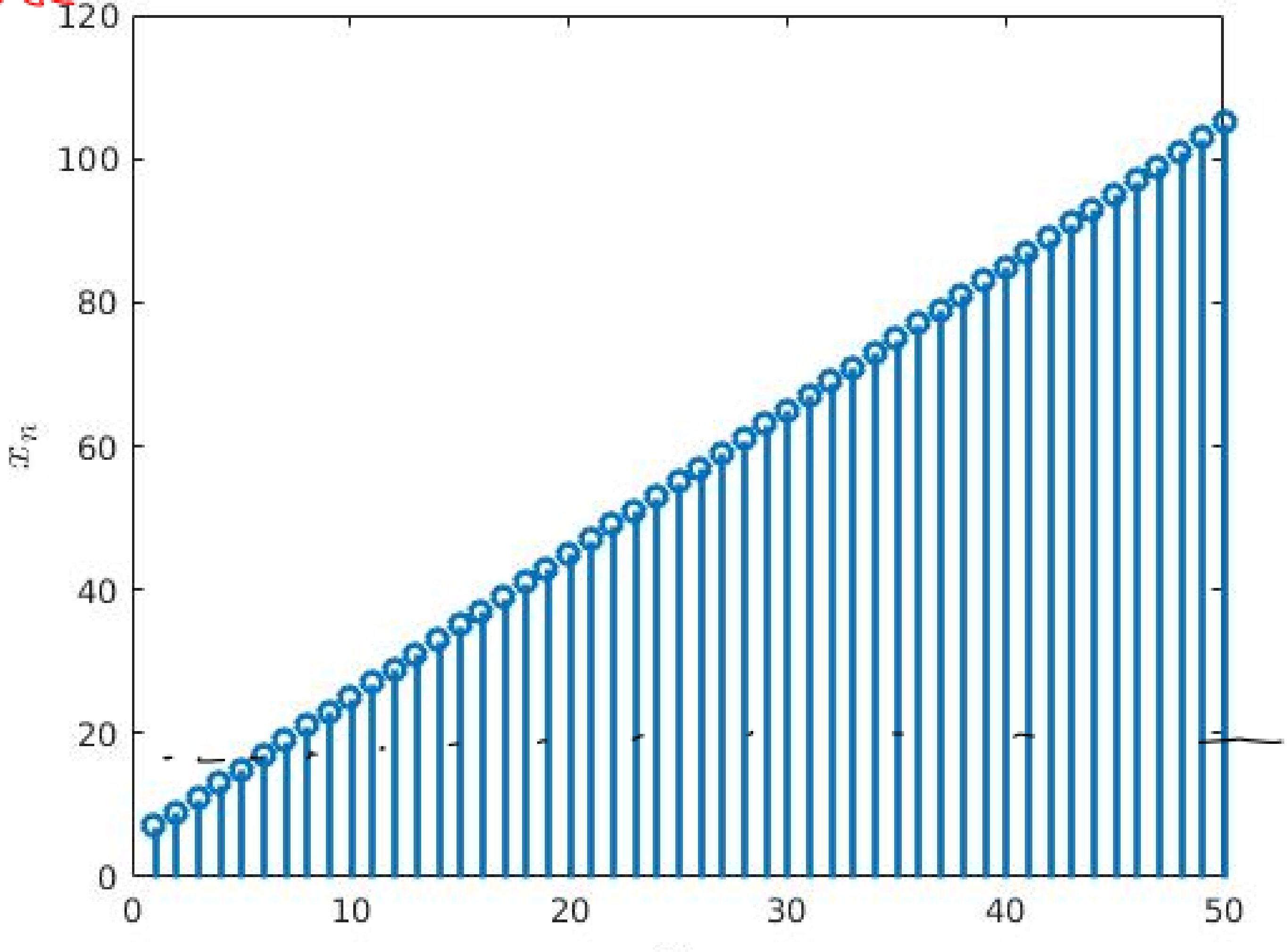
another open interval centred at $L=0$

all but 3 terms are contained in this interval

CONVERGENT SEQUENCE

DIVERGENT SEQUENCE

$\{x_n\}_{n\in\mathbb{N}}$ with $x_n = 2n+5$



consider the open interval $(-20,20)$. Infinitely many terms are not contained in this interval.

Example: 1 (a) $\left\{ \frac{3n+1}{7n-4} \right\}_{n \in \mathbb{N}}$

Rewrite as $x_n = \frac{\cancel{n}(3 + 1/n)}{\cancel{n}(7 - 4/n)} = \frac{3 + 1/n}{7 - 4/n}$

as $n \to \infty$, $1/n \to 0$ and $4/n \to 0$

hence as $n \to \infty$, $x_n \to \frac{3}{7}$

Yes, this is convergent sequence with $L = 3/7$.