#1.1 Sequences

LECTURE 1 1/10/2017

A sequence is an ordered list of objects (typically real numbers) called terms.

Def^n (Real sequence)

If for every positive integer n, there is associated a real number a_n, then the ordered set

$$a_1, a_2, a_3, \ldots, a_n, \ldots.$$

is said to define an infinite sequence.

Note:

- each member of the set has been labeled with an integer. So we can speak of the first term a_1, the second term a_2, and, in general, the n^{th} term a_n
- each term a_n has a successor a_{n+1} and hence there is no "last" term.

Describing Sequences

① **Direct Formula** A sequence can be described if we give a rule or formula for the n^{th} term.

Example: the formula $a_n = \frac{1}{n}$ defines a sequence whose first few terms are $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$.

Sometimes, two or more formulas can be used; for example

$$a_{2n-1} = 1, \quad a_{2n} = 2n^2$$

the first few terms of this sequence is $1, 2, 1, 8, 1, 18, 1, 32, 1, \ldots$.

$a_1\ a_2\ a_3 \ldots$

② **Recursive Formula**

Another common way to define a sequence is by a set of instructions which explains how to carry on after a given start.

Consider $a_1 = a_2 = 1, \quad a_{n+1} = a_n + a_{n-1}$ for $n \geq 2$.

This rule defines a famous sequence whose terms are called Fibonacci numbers

the first few terms are

$1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$

Notation For brevity, the notation $\{x_n\}$ (or $\{x_n\}_{n \in \mathbb{N}}$ or $\{x_n\}_{n \geq 1}$) to denote a sequence whose n^{th} term is x_n

Examples ① Determine a recursive formula for a sequence whose first few terms are

ⓐ 4, 10, 16, 22, 28, ...

(first term) $y_1 = 4$

$y_{n+1} = y_n + 6$ if $n \geq 1$.

Why? the difference b/w two successive terms is 6

ⓑ 3, 6, 9, 15, 24, 39, 63, ...

looks like each term is the sum of two previous terms

$x_1 = 3$, $x_2 = 6$, and $x_{n+1} = x_n + x_{n-1}$ for $n \geq 2$.

② Determine a direct formula for a sequence whose first few terms are

ⓐ 50, 48, 46, 44, 42, ...

(subtract 2 to obtain next term)

$x_n = 50 - 2(n-1)$, $n \geq 1$.

ⓑ 4096, 2048, 1024, 512, 256, ...

(divide each term by 2 to obtain successor)

$x_n = 4096 \left(\frac{1}{2}\right)^{n-1}$ for $n \geq 1$.