Edge Detection from Spectral Data with Application to PSF Estimation

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Introduction

Detection of jump discontinuities constitutes an important task in several areas of engineering and signal processing. In certain applications such as MR imaging, input data is collected in the spectral domain. In such cases, the concentration method allows us to compute jump locations and values from a finite number of spectral coefficients. In this paper:

- We discuss the design of concentration factors, which determine the characteristics of the jump approximation.
- We provide waveform-aware iterative formulations of the method for accurate jump detection.
- We discuss an application of the method to estimating point-spread functions (psf) in blurring problems.

The Concentration Method

Let \( f \) be a 2π-periodic piecewise-smooth function. We define its jump function as

\[
[f](x) = f(x^+) - f(x^-)
\]

Note that the jump function is non-zero only at the singular support of \( f \). Given the Fourier coefficients \( \hat{f}(k) \) \( k \in \mathbb{Z} \), the concentration method, [1,2], computes an approximation to the jump function using the following partial sum

\[
S_N[f](x) = \frac{1}{2\pi} \sum_{k=-N}^{N} \hat{f}(k) \sin(kx) \frac{\sigma(\frac{k}{N})}{\sigma(k)} e^{ikx}
\]

where \( \sigma(x) = \sigma(\frac{x}{N}) \) are known as concentration factors. These factors are known to satisfy certain admissibility conditions:

- \( \sigma(x) \) is odd
- \( \sigma(x) \in C^0(0,1) \)
- \( \frac{\sigma(x)}{\sigma(\tau)} \to -\pi, \tau = \epsilon N > 0 \) being small

Iterative “Waveform-aware” Edge Detection

- We take inspiration from sparsity enforcing regularization routines and their iterative solutions. [3]
- We exploit the characteristic responses of the concentration factors

\[
S_N[f](x) = \left[ f * W_N^K \right](x) \approx \left[ f * W_N \right](x)
\]

Here, \( W_N^K \) is the characteristic response to a unit jump.

Iterative Problem Formulation

\[
\min \| W p - S_N[f] \|_2^2 + \lambda \| p \|_1
\]

where \( W \) is a toeplitz matrix built out of the characteristic response.

Concentration Factor Design

Integrating the Fourier integral by parts, we obtain

\[
\hat{f}(k) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[ f(x^+) - f(x^-) \right] \sin(kx) \frac{\sigma(\frac{k}{N})}{\sigma(k)} e^{ikx} \, dx
\]

where \( \zeta, a \in \mathbb{A} \) denote the jump locations. Substituting this expression in the concentration sum, we obtain

\[
S_N[f](x) = \sum_{k=1}^{N} \hat{f}(k) \sin(kx) \frac{\sigma(\frac{k}{N})}{\sigma(k)} e^{ikx}
\]

with \( D_N^\sigma(x) = 1 \sum_{k=-N}^{N} \hat{f}(k) \sin(kx) \frac{\sigma(\frac{k}{N})}{\sigma(k)} e^{ikx} \), \( v \geq 0, 1, 2, \ldots \)

- Design Objective: enhance \( D_N^\sigma(x) \), reduce \( D_N^\sigma(x), p > 0 \)

Sample Problem Formulations

- **P1**: \( \min \| D_N^\sigma - \delta \|_1 \) subject to \( D_N^\sigma \geq 1 \)

- **P2**: \( \min \| D_N^\sigma - \delta \|_1 \) subject to \( D_N^\sigma \leq 10^{-4}, D_N^\sigma \leq 10^{-4}, \sigma \geq 0 \)

- **P3**: \( \min \| D_N^\sigma - \delta \|_1 \) subject to \( D_N^\sigma \geq 1, D_N^\sigma \leq 10^{-4}, \sigma \geq 0, \sigma(1) = 0, \sigma(K) = 0, \| D_N^\sigma \|_{\infty} \leq 10^{-4}, \) for \( K \subset [-N,N] \)

Consider the convolutional blurring model

\[
g = f * h + n
\]

where \( f \) is the true function, \( h \) is the unknown blur or the point-spread function (psf), \( n \) is noise and \( g \) is the observed function. Let us apply the concentration edge detector.

We have:

\[
S_N[g] = T (f * h + n) = (f * h + n) * K_N^\sigma = f * h * K_N^\sigma + n * K_N^\sigma = (f * K_N^\sigma) * h + n * K_N^\sigma = f * h + n * K_N^\sigma
\]

Hence, we observe shifted and scaled replicates of the psf.

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