No calculators, no notes, no books. Only pens, pencils and erasers are allowed.

1. Say for each of the following polynomials if it is reducible or irreducible:

(a) \( x^2 + 1 \) in \( \mathbb{R}[x] \).
   It is irreducible, because it is of degree 2 and has no root in \( \mathbb{R} \)

(b) \( x^2 + 1 \) in \( \mathbb{Z}_2[x] \).
   It is reducible because it has a root \( x = 1 \).

(c) \( x^2 + 1 \) in \( \mathbb{Z}_3[x] \).
   It is irreducible because it does not have a root in \( \mathbb{Z}_3 \) and it is of degree 2.

2. Find the inverse of \( x + 1 \) in \( \mathbb{Z}_2[x]/(x^2 + x + 1) \).

\[
x^2 + x + 1 = x(x + 1) + 1
\]

\[
1 = x^2 + x + 1 + x(x + 1)
\]

so the inverse of \( x + 1 \) is \( x \).