1. Say for each of the following statements whether it is true or false (without explanation):

   a. $\mathbb{Z}$ is an integral domain.
      True.

   b. $\mathbb{Z}_6$ is an integral domain.
      False. (It contains zero divisors: $[2] \cdot [3] = [0]$.)

   c. $\mathbb{R}[x]$ is a field.
      False. (Not every element is invertible, e.g. $x$.)

   d. $M_2(\mathbb{R})$ is a field.
      False. (There are zero divisors: \[
      \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.\])

   e. In $\mathbb{Z}$, $5 | 15$.
      True.

   f. In $\mathbb{Q}$, $15 | 5$.
      True. ($5 = 15 \cdot \frac{1}{3}$ and $\frac{1}{3} \in \mathbb{Q}$.)

   g. In $\mathbb{Z}[x]$, $2x | x^2$.
      False.

   h. In $\mathbb{R}[x]$, $1 + x^2 + 5x^6 \equiv 2 + x^2 + 5x^6$ (mod $x$).
      False. (The difference between the polynomials is 1 which is not a multiple of $x$.)

2. Prove: In a commutative ring $R$, if $c|a$ and $c|b$ then $c|ab$.
   
   Proof: Since $c|a$, $a = rc$ for some integer $r$. Since $c|b$, $b = sc$ for some integer $s$. Therefore $ab = (rc)(sc) = (rsc)c$, which means $c|ab$. 