Math 310-03  Quiz 2  September 18, 2015

No calculators, no notes, no books. Only pens, pencils and erasers are allowed.

1. Consider the following relations on $\mathbb{N} \setminus \{0, 1\}$:
   - $aR_1 b \iff a|b$. [$a|b$ means $a$ divides $b$]
   - $aR_2 b \iff \gcd(a, b) > 1$. [$\gcd(a, b) > 1$ means that there exists an integer greater than 1 dividing both $a$ and $b$]

   Fill in yes or no in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Reflexive</th>
<th>Symmetric</th>
<th>Transitive</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>$R_2$</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

   (Use the space below if you want, but only the final answer in the table takes into account.)

   For the no’s: We have 2 | 4 but not 4 | 2. We have gcd(6, 9) > 1 and gcd(4, 6) > 1, but gcd(4, 9) = 1.

2. Consider the functions $\mathbb{N} \to \mathbb{N}$
   - $f(x) = 2x + 1$.
   - $g(x) = \lfloor \frac{x}{2} \rfloor$. [For example, $g(3) = 1$, $g(4) = 2$, $g(5) = 2$]

   Say about the following functions whether they are bijections or not. If they are, give a formula. If not, explain why:
   - $f \circ g$.
     This is not a bijection. For instance, $f \circ g(4) = 5 = f \circ g(5)$.
   - $g \circ f$.
     This is a bijection. The formula is $g \circ f(x) = g(f(x)) = g(2x + 1) = x$. 