Math 310-01          Quiz 2          September 18, 2015

No calculators, no notes, no books. Only pens, pencils and erasers are allowed.

1. Consider the following relations on \( \mathbb{R} \):
   
   - \( aR_1 b \iff |a - b| \leq 1 \).
   - \( aR_2 b \iff a - b < 1 \).

   Fill in yes or no in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Reflexive</th>
<th>Symmetric</th>
<th>Transitive</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1 )</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>( R_2 )</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

   (Use the space below if you want, but only the final answer in the table takes into account.)
   For the no’s: We have \( 2 - 5 < 1 \) but not \( 5 - 2 < 1 \). We have \( |2 - 1| \leq 1 \) and \( |3 - 2| \leq 1 \) but not \( |3 - 1| \leq 1 \). We have \( 1 - \frac{1}{2} < 1 \) and \( \frac{1}{2} - 0 < 1 \) but not \( 1 - 0 < 1 \).

2. Consider the functions \( \mathbb{N} \to \mathbb{N} \)

   - \( f(x) = 2x \).
   - \( g(x) = \lfloor \frac{x}{2} \rfloor \). [For example, \( g(3) = 1, g(4) = 2, g(5) = 2 \)]

   Say about the following functions whether they are bijections or not. If they are, give a formula. If not, explain why:

   - \( f \circ g \).
     This is not a bijection. For example, \( f \circ g(4) = 4 = f \circ g(5) \).

   - \( g \circ f \).
     This is a bijection. The formula is \( g \circ f(x) = g(f(x)) = g(2x) = x \).