Exercise.

- Let $R$ and $S$ be two integral domains. Is $R \times S$ always an integral domain, sometimes or never?
  Solution: Never. One can always look at $(1, 0)$ and $(0, 1)$. They are nonzero and their product is the zero element $(0, 0)$, so they are zero divisors and $R \times S$ is not an integral domain.

- Let $S$ be a subring of an integral domain $R$. Assume $S$ contains 1. Is $S$ necessarily an integral domain?
  Solution: Yes. Since $S$ is a subring of $R$, it must also be commutative, and it is assumed to contain 1. Every two nonzero elements in $S$ are also nonzero elements of $R$, so their product is nonzero because $R$ is an integral domain. Therefore $S$ contains no zero divisors, hence it is an integral domain.

- Let $S$ be a subring of a field $R$. Assume $S$ contains 1. Is $S$ necessarily a field?
  Solution: No. Take $R = \mathbb{Q}$ and $S = \mathbb{Z}$. $S$ is a subring of $R$ containing 1, but it is not a field.