Home Exercise.
Find $L_1 + L_2$ and $L_1 \cap L_2$ in the following cases:

- $R = \mathbb{Z}_{30}, L_1 = 4\mathbb{Z}_{30}, L_2 = 2\mathbb{Z}_{30}$.
- $R = \mathbb{Z}_{30}, L_1 = 4\mathbb{Z}_{30}, L_2 = 3\mathbb{Z}_{30}$.
- $R = \mathbb{Z}_{30}, L_1 = 4\mathbb{Z}_{30}, L_2 = 6\mathbb{Z}_{30}$.

Solution.

- Notice that $4\mathbb{Z}_{30}$ is the same as $2\mathbb{Z}_{30}$, so $L_1 \cap L_2 = L_1 + L_2 = L_2 = 2\mathbb{Z}_{30}$.
- Recall that $L_1 = 2\mathbb{Z}_{30}$. Then $L_1 + L_2 = \mathbb{Z}_{30}$ and $L_1 \cap L_2 = 6\mathbb{Z}_{30}$.
- $L_1 \supseteq L_2$, so $L_1 + L_2 = L_1 = 2\mathbb{Z}_{30}$ and $L_1 \cap L_2 = L_2 = 6\mathbb{Z}_{30}$. 