Exercise.
Let $f : R \to S$ be a homomorphism. Show that $\text{Im}(f)$ is a subring of $S$.

Solution.
First, $f(0) = 0_S$, so $0_S \in \text{Im}(f)$. Let $a, b \in \text{Im}(f)$. Then there exist $a', b' \in R$ such that $f(a') = a$ and $f(b') = b$. Now, $f(a' + b') = f(a') + f(b') = a + b \in \text{Im}(f)$. Similarly $f(a'b') = ab \in \text{Im}(f)$, and $f(-a') = -f(a') = -a \in \text{Im}(f)$. 